FROM SQUARE TO HYPERHYPERCUBE

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Word Squares

A word square is like a crossword puzzle; a set of words is used to fill in the square both horizontally and vertically. Unlike a crossword puzzle, though, a word square has no blacked-out portions. If the vertical words are identical with the horizontal words, then the word square is said to be regular. Here is an example of a regular word square:

F A C E D
A L I V E
C I V I L
E V I C T
D E L T A

Of course, there is no reason why the vertical words should be identical to the horizontal ones. When the two subsets of words do differ, the word square is termed a double word square. An example:

A P P L E
R E L A X
O R A T E
M I N E R
A L E R T

For use later on in this article, let us introduce some mathematical terminology. A word square of size 5 x 5 is said to have side 5. In general, a word square of size a x a (where _a_ can be any number) is said to have side _a_. It isn't difficult to see that an a x a word square has a total of a^2 letters and 2a words in it. In a double word square there will be 2a different words, while in a regular word square there will be _a_ different words, each being used twice.

Word Cubes

Word cubes are just the extension of word squares into a third dimension. Not only can words be read vertically and horizontally, but they can also be read in a mutually perpendicular direction, the third dimension. Just as we had regular and double word squares, analogously we have regular and <u>triple</u> word cubes.

For example, consider the regular word cube below:

C A R T	A V E R	R E N O	T R O T
A V E R	V I V A	E V E R	R A R E
R E N O	E V E R	N E S T	O R T S
T R O T	R A R E	O R T S	T E S T

The words in the third dimension are read straight off from each of the smaller squares. There are sixteen of these. The across words are read by taking a letter from a particular position in each of the squares of a row. The down words are read by taking a letter from a particular position in each of the squares of a column. For example, let us take the third row of squares in the above diagram. Take the second letter in each of the four squares in this row. The letters thus selected are E, V, E and R, which make the word EVER. In all, there are sixteen of these across words; similarly, there are sixteen down words.

The perceptive reader will note that four of the words occur only three times -- those in the squares of the top-left/bottom-right diagonal in the diagram (this is the so-called leading diagonal). The four words are CART, VIVA, NEST and TEST. The other six words are used six times each -- these are AVER, EVER, ORTS, RARE, RENO and TROT. Four words used three times each and six words used six times each account for the forty-eight words in the cube. In a regular word cube of size a x a x a, there will be <u>a</u> words used 3 times each, and a(a - 1)/2 words used 6 times each.

The reader may care to inspect a triple word cube which we have constructed:

M O N A	A P O D	L A I D	I L L S
I R A N	C O L E	E L M A	S E A T
T E S T	E L L A	A S A R	M E T E
A S T I	R O A D	T O M E	S T E M

The sixteen across words in this cube are: MALI, OPAL, NOIL, ADDS, ICES, ROLE, ALMA, NEAT, TEAM, ELSE, SLAT, TARE, ARTS, SOOT, TAME and IDEM. The sixteen down words are: MITA, ORES, NAST, ANTI, ACER, POLO, OLLA, DEAD, LEAT, ALSO, IMAM, DARE, ISMS, LEET, LATE and STEM. The sixteen words in the third dimension are: MONA, APOD, LAID, ILLS, IRAN, COLE, ELMA, SEAT, TEST, ELLA, ASAR, METE, ASTI, ROAD, TOME and STEM. In general, we note that a triple word cube of size a x a x a has a total of a^3 letters and $3a^2$ words in it. The above triple word cube is imperfect, as the word STEM appears in two different places.

Word Hypercubes

As far as we know, no one has taken the construction of word forms beyond the third dimension. We, however, in true pioneering spirit have ventured into the fourth dimension. The fourth dimension that we are concerned with is not time, but is a fourth spatial dimension. Don't think that there is anything at all mystical about a fourth dimension (or a fifth one, or a sixth, and so on). Mathematicians often find it convenient to assume a space of n dimensions and infer the characteristics of geometric figures in such a space.

This four-dimensional structure we are going to call a hypercube. When we insert letters into its unit hypercubes, we will arrive not at a word square or a word cube, but at a word hypercube. Just as a word cube can be represented in two dimensions, so too can a word hypercube.

We have not bothered to construct a regular word hypercube where some words are used more than once. Instead, we have plunged straight into the incredibly difficult task of building a quadruple word hypercube (analogous to a triple word cube and a double word square). A quadruple word hypercube of side 3 will contain 108 different three-letter words. Here is our example:

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A L A   R O B   T W O
A E N   T E U   A R N
R A A   A R M   E Y E

E A N   I B A   E A R
S R I   Y A S   R I E
E A S   O Y E   S A W

S O N   A E A   T S T
H A E   E T H   O I I
A M P   R E U   S L E
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The large square is made up of nine smaller squares, each of these smaller squares being a 3 x 3 word square in its own right. These nine squares account for 54 words, exactly one-half of the 108 words that must appear in the hypercube. How do we get the other 54 words? Take any one of the three rows or three columns of 3 x 3 squares. From the three squares selected that constitute a row or a column, choose a letter in the same position from each of the three squares. These three letters will together form a word.

Let us give the reader an example. Suppose we select the second row of 3 x 3 squares:

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E A N   I B A   E A R
S R I   Y A S   R I E
E A S   O Y E   S A W
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We now choose a letter in the same position from each of these three squares. Suppose we choose those from the bottom right-hand corner of each square. The letters chosen are S, E and W, which make the word SEW. Since the bottom right-hand corner is one out of nine possible starting positions, it is evident that the second row of 3 x 3 squares given above yields a total of nine words. Similarly, one obtains nine words from the first row of 3 x 3 squares, and nine more from the third row of 3 x 3 squares, for a total of 27 words. The remaining 27 words are obtained by initially selecting one of the three <u>columns</u> of 3 x 3 squares.

No word is used twice in this hypercube. While most of the words are uncommon, they can all be found in the Second and/or Third

Editions of Webster's New International Dictionary. A complete list of all the 108 words in the hypercube is given below.

AAM	ANN	AYA	ESE	MEL	OII	RII	TAE
AAR	AOR	BAA	ESS	MEU	ONE	ROB	TET
ABA	ARA	BAY	ETE	NAR	ORT	RTA	TEU
ABO	ARM	BUM	ETH	NAT	OYE	RYE	TIE
AEA	ARN	EAN	EWE	NEI	PUE	SAT	TOS
AEN	ARO	EAR	EYE	NEP	RAA	SAW	TST
AER	ARS	EAS	HAE	NIE	RAE	SEW	TWO
AES	ART	EAT	HEO	NIS	RAI	SHA	TYE
AHU	ARY	EER	IBA	NUN	REA	SIL	USH
AIA	ASE	EHI	ISE	OAM	REU	SLE	WAS
ALA	ASH	EIE	IYO	OBE	REW	SON	WRY
AME	ASP	EOS	LAO	OER	RIA	SRI	YAL
AMP	ATA	ERA	LEA	OES	RIE	SYR	YAS
ANA	ATI	ERS	LOW				

In general, a quadruple word hypercube of side $\underline{a}$ has a total of a^4 letters and $4a^3$ words.

Word Hyperhypercubes

We define a hyperhypercube as one having five dimensions. If we attempt to construct one with side 3 (three letters per word), we will have to use 243 letters to form a total of 405 words! We feel that the construction of a hyperhypercube with this many words in it would be far too time-consuming, and leave the task to our electronic friend, the computer.

But -- suppose the words in our hyperhypercube have only two letters each. Would our task be that much easier? A 2 x 2 x 2 x 2 x 2 structure would consist of 32 letters forming 80 words. The task of constructing such a word hyperhypercube doesn't appear to be too daunting, even when we add the stipulation that all 80 words are to be different. Anyone care to tackle the task?