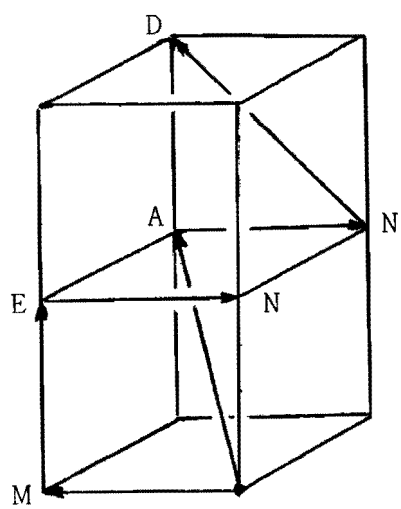
WORD WORMS

A. ROSS ECKLER
Morristown, New Jersey

In a 3-by-3-by-3 lattice there are 27 cubes of equal size - a central cube surrounded by 26 neighbors. Assign the 26 letters of the alphabet to these cubes according to the following diagram:

Top Layer	Middle Layer	Bottom Layer
A B C	J K L	R S T
D E F	M . N	U V W
G H I	O P Q	X Y Z

Using this array as a template, one can trace out words in three-dimensional space. To form the word AND, for example, first draw a line from the dot in the central cube to the center of the cube in the upper left corner, containing A. Next, move the lattice, without twisting or turning it, so that the central cube is located where the A-cube was, and draw a line from the (relocated) dot to the center of the cube containing N. Move the lattice again so that the central cube is located where the N-cube was, and draw a line from the dot to the center of the cube containing D. These three lines form a **word worm**, as shown in the picture below, which also depicts the word worm corresponding to MEN.



If the sides of the cubes are of unit length, the word worm corresponding to AND consists of a line of the square root of 3 in length, followed (at an angle of 60 degrees) by a line of unit length, followed (at an angle of 45 degrees) by a line of the square root of 2 in length. (From now on, all lines will be identi-

fied by their squared lengths.)

Word worms evolve in an amazing variety of ways; in fact, any word worm of four or more letters has a shape that is almost certainly unique. This article develops a taxonomy of worms, emphasizing certain shapes of special interest. To begin with, one-letter worms sort into three straight-line worms of different lengths: ACGIRTXZ with (squared) length 3, BDFHJLOQSUWY with (squared) length 2, and EKMNPV with length 1. Two-letter words generate a slightly more interesting class of word worms - one (AA) is still a straight line, but longer; most are bent; and a few (BY, LO, SH) fold back upon themselves. In fact, there are 28 varieties, all classified in the Appendix along with the 676 letter-pairs creating them. (Capitalized letter-pairs correspond to two-letter words admissible in the game of Scrabble, as listed by the OSPD.)

When one considers three-letter words, the worms become far more interesting. To begin with, there are 728 different kinds; the 28 varieties in the Appendix can be augmented by a third segment projected in the 26 different directions defined by the template. Of these, a few form closed loops, with the head of the worm seizing its own tail. These I call Ouroboros worms in honor of the 1922 science-fiction story by E.R. Eddison, The Worm Ouroboros, in which the Worm of Time grasps its own tail in its mouth (in Greek, **oura** means "tail" and **bora**, "food"). It is a straightforward task to ask the computer to find those words that generate Ouroboros worms. Assign each letter of the alphabet a vector according to its position in the lattice (A = 1,-1,1; B = 1,0,1; C = 1,1,1; ... , Z = -1,1,-1), and add up the vectors of the letters in the word; if the sum is (0,0,0) the worm has bitten its tail.

There are, in fact, only a few varieties of three-letter Ouroboros worms: the 30-60 right triangle with sides 1,2,3; the isosceles triangle with sides 1,1,2; the equilateral triangle with sides 2,2,2. (Remember, all lengths are given in squared units.) It is geometrically impossible for an equilateral worm to have sides 3,3,3 or 1,1,1, as a quick check of the vectors will reveal. Among OSPD words, the right triangle is represented by ANY, CUP, ISM, LEX, MIS, NAY, PAW, SIM, TOE and WAP; the isosceles, by KEY, MEW and PES; and the equilateral, by BOW. There are a few OSPD three-letter words that make foldback straight-line worms (EVE, VEE, LOO, SHH) or foldback bent worms (AZO, BYE, BYS, FUB, BUD, FUG, FUN, LOB, LOG, LOP, LOT, LOW, LOX, OLD, OLE, VEG, VET, VEX, ZAP, ZAX).

Technically, the folded worms corresponding to two-letter words such as BY are also Ouroboros worms, since they bite their own tail; however, I prefer to reserve the term Ouroboros for those worms which only bite their tail, not fold as well. The line between Ouroboros worms and folded worms, however, is not easy to draw, as will be seen when longer words are classified.

Four-letter words generate new taxonomic specimens. The most interesting, perhaps, is the non-planar Ouroboros worm - one that is not confined to a single plane in 3-dimensional space,

as all three-letter Ouroboros worms must be. And, for the first time, one must be on the lookout for worms that intersect themselves at isolated points other than their tails - say, at the end of one line where it joins the next, or even in the middle of a line.

The OSPD provides examples for many different four-letter worm types. However, a theoretical total of 18,928 = 18x16x26 can be distinguished, suggesting that two four-letter words with the same geometric configuration must not be easy to find (any candidates?). Most words correspond to relatively uninteresting zig-zaggy non-intersecting worms; I concentrate on the exceptions. The Ouroboros worms come in numerous varieties. It is theoretically possible for a square worm to exist, but none have been found (patterns such as JOQL, BHYS or FWUD would have sides of length 2, and KMPN, KEPV or MENV would have sides of length 1). However, ROIL is a rectangle with sides 2,3,2,3, FOUL is a rhomboid with internal angles of 60 and 120 degrees and sides 2,2,2,2, and VIER is a rhomboid with internal angles of 60 and 120 degrees and sides 1,3,1,3. Finally, COOT is an isosceles triangle with sides of length 3 and a base of length 4.

Foldback worms come in more varieties than previously. One can have those that consist of two lines first generated and then folded back, as in VOLE and LEVO (a line of length 1, followed by a line of length 2 at 90 degrees), BEVY (line of length 2, followed by a line of length 1 at 135 degrees), WOLD (line of length 2, followed by a line of length 2 at 60 degrees), or GIRT, TRIG, TRIG and IZAR (line of length 3, followed by a line of length 3 at 120 degrees). Or, one can have a worm which folds back twice to its tail, as in LOVE (a line of length 2 folded back upon itself, followed by a line of length 1 at an angle of 30 degrees also folded back on itself) or SHRI (a line of length 2 folded back upon itself, followed by a line of length 3 at an angle of 30 degrees folded back upon itself). One might observe that LOVE is a rather spiky experience!

It is now time to examine the varieties of Ouroboros worms more closely. As mentioned earlier, I have excluded worms such as BY or BEVY which have lines folding back upon themselves. (Even a single foldback in an otherwise-conventional Ouroboros worm, such as SH in SHIMS, is a disqualification.) On the other hand, I admit as Ouroboros those worms which internally intersect at isolated points, either line-ends or line-centers. Two examples of the latter phenomenon are EYES and VIVA, both of which look like bow ties. In EYES, the two E-lines are parallel, and the Y-line and S-line connect opposite ends of the E-lines, themselves intersecting (at a 90 degree angle) at their centers. An example of the former phenomenon is MISPLAY, consisting of a triangle, MIS, followed by a quadrilateral, PLAY. I call pure Ouroboros worms those that do not have any crossings of either type; such worms truly form a loop in 3-dimensional space.

The most elegant nonplanar Ouroboros worm is clearly TAXI, which traces out two-thirds of the edges of a tetrahedron: four lines of length 3, each angled at 60 degrees with respect to its neigh-

bers. There are a number of isosceles relatives to TAXI, i.e., words which fold along a line joining two opposite corners of the Ouroboros worm, and which consist of two equal-length lines on either side of the fold:

PULE 90 degree fold; lengths 1,1,2,2
WOKE HUNK NOSE ONES SONE 45 degree fold; lengths 1,1,2,2
DUIT 30 degree fold; lengths 2,2,3,3

Also, there are a few non-planar Ouroboros worms of irregular shape, with line lengths of 1,2,2,3, of 1,2,3,2, and of 1,3,2,2:

ZEDS 3(60)1(135)2(60)2(90)
ZEBU 3(60)1(135)2(60)2(90)

SCOP 2(90)3(30)2(135)1(45)
COPS 3(30)2(135)1(45)2(90)
PLAY 1(45)2(90)3(30)2(135)

LUNG 2(60)2(45)1(60)1(30)

PALY 1(60)3(150)2(60)2(135)

UNCO 2(45)1(120)3(30)2(120)

Finally, a variation of the folding theme is provided by the words ELMY and EONS (or NOES). These consist of two right triangles of sides 1,2,3 folded at a 45 degree angle with respect to each other along their hypotenuses.

For words of five letters or more, a full taxonomy of Ouroboros worms is even more complex, However, it is worth looking at six-letter words at least briefly because of their high potential for symmetrical Ouroboros worms. Hexagons with sides of either 2 or 3 are theoretically possible. No words forming hexagonal Ouroboros worms are known, but TRAGIC is a (tragic!) near-miss. Since TRAGIZ forms a hexagon, TRAGIC can be viewed as a bracelet with an open clasp.

Turning to foldback worms, one has the triple spike examples of SHRIVE and EVOLVE, as well as the three-line out-and-back example of WIZARD, and the double spikes SHIVER, SHOVEL, and REVIVE. HYBRIS is a Y-shaped foldback worm.

A list of OSPD Ouroboros worms of five to eleven letters is given in the Appendix. In addition, I give a list of boldface Webster Second or Third Edition words of 14 letters or longer that form Ouroboros worms. As can be seen from the table below, Ouroboros worms form an increasingly-small percentage of words as word length increases.

Length	Percentage	Length	Percentage
3	1.54	9	0.23
4	0.73	.	
5	0.53	.	
6	0.31	14	0.17
7	0.28	15	0.05
8	0.23	16	0.09

he construction of a word worm can be likened to a random k on a lattice; at each intersection, one throws a 25-sided (not 26, because an Ouroboros worm doesn't permit doubling k on the same line) to decide where to go next. In a 1940 er published in the Proceedings of the Royal Society of Edin-gh, mathematicians McCrea and Whipple proved that if one ted at any point on an infinitely-large 3-dimensional lattice six paths emanating from each intersection, there is a prob-ity of 0.35 that a random walk will eventually come back to point. With more than four times as many exit paths available each intersection, and a limit on the number of steps taken word length), it is hardly surprising that the chance of gen-ing an Ouroboros worm is so small. The chance is larger for words only because the start is still quite near.

word length increases, pure Ouroboros worms - those that ain no intermediate intersections - become increasingly rare. e are two Websterian words of 15 letters, TRYPANORHYNCHAN SEMICONSPICUOUS, and five 14-letter ones, HERNIOPUNCTURE, NTIMONIOUS, SEMIBITUMINOUS, ULTIMOGENITARY and HYSTERO-CIUM.

e other class of worms deserves mention: those that never rsect themselves. Such worms are in the overwhelming majority short word-lengths; even for Websterian words of 20 letters more, they form about 0.30 of the total. The longest-known of this nature is the 27-letter ETHYLENEDIAMINETETRAACETATE.

open question: what word worm ends up farthest from the ? Which worm at some point in its segmented length is farth-from the start? Both are likely to be long words, of course. er open question: what is the longest word worm which has in" - another word with the same shape?

of the pure Ouroboros worms in this article are topologically valent to a ring. Is it possible to find a pure Ouroboros worm is topologically equivalent to a simple overhand knot? A little -and-error soon convinces one that the minimum number of needed to tie a worm into a knot is nine; an example of ord" which creates such a knot is TYDBNYRDI. It is quite for most people to visualize knottedness without a model; ggest constructing a lattice out of Tinkertoy and threading ring through it, or, on a larger scale, using a rope on an shioned school playground Jungle Gym. In view of the rarity ords of nine or more letters that form pure Ouroboros worms, ems exceedingly unlikely that a knotted worm can be found. amming a computer to ascertain whether or not an Ouroboros has a knot in it looks like a difficult task.

nally, a philosophical question: is the assignment of letters utside cubes in the 3-by-3-by-3 lattice the most "natural" I can conceive of others, such as the one below:

G H I	L K J	X Y Z
F E D	M . N.	W V U
A B C	Q P O	R S T

This 3-dimensional boustrophedon pattern preserves the AZ, BY, ... MN symmetries. Note that the letters are still classified into the same line-lengths; in fact, it appears that one will end up with words classified into the same taxonomy!

The concept of a word worm was originally developed by participants of "Words Forum" on the IBMTEXT computer bulletin board. On July 7, 1992, Keith Jones (WINVMJ) suggested the idea of labelling the 26 outside cubes in a 3-by-3-by-3 lattice with the letters of the alphabet ("an incredibly silly idea ... [but] sometimes the most idiotic notions blossom into full-scented ... lunacy"); he also proposed (among others) the name **worms**. The idea of using the lattice as a template, leading to worms extending beyond the original lattice, is due to Grant Willson (UITVM1), responding three days later. Philip Cohen passed the information along to me. My son-in-law, Tom Day, provided programming help and proposed the name Ouroboros for worms that bite their tails.

APPENDIX

Letters of length 3: ACGIRTXZ Letters of length 2: BDFHJLOQSUWY
Letters of length 1: EKMNPV

3(180)3 AA cc gg ii rr tt xx zz
3(120)3 ac ag AR ca ci ct ga gi gx ic ig iz ra rx rt tc tr tz xg xr xz zi zt zx
3(60)3 AI AX AT cg cr cz gc gr gz ia IT ix rc rg rz TA TI tx xa XI xt zc zg zr
3(0)3 az cx gt ir ri tg xc za

3(150)2 ab AD aj cb cf cl gd gh GO IF ih iq rj rs ru tl ts tw xo XU xy zq zw zy
3(90)2 af AH al ao AS au cd ch cj cq cs cw gb gf gj gq gu gy ib ID il io iw iy
rb rd rl ro rw ry tb tf tj tq tu ty xd xh xj xq xs xw zf zh zl zo zs zu
3(30)2 aq AW AY co cu cy gl gs gw ij IS iu rf rh rq td th TO xb xf xl zb zd zj

3(120)1 AE ak am ce ck cn ge gm gp ie IN ip rk rm rv tk tn tv xm xp xv zv zn zp
3(60)1 AN ap av cm cp cv gk gn gt ik im iv RE rn rp te tm tp xe xk xn ze zk zm

2(150)3 BA bc DA dg fc fi hg HI ja jr og OX qi qz lc lt ur ux sr st wt wz yx yz
2(90)3 bg BI br bt dc di dr dx FA fg ft fz HA hc hx hz jc jg jt jx LA LI lr lz
oa oi OR oz qc qg qt qx sa sc sx sz ua ug UT uz wc wi wr wx yg yi yr yt
2(30)3 bx bz dt dz fr fx hr ht ji jz lg lx oc ot qa qr sg SI uc ui wa wg YA yc

2(180)2 bb dd ff hh jj ll oo qq ss uu ww yy
2(120)2 bd bf bj bl db dh dj DO fb fh fl fq hd hf HO hq jb jd js ju lb lf ls lw
OD OH ou OY qf qh qw qy sj sl su sw uj uo US uy wl wq ws wy yo yq yu yw
2(90)2 bh bs df du fd fw hb hy jl JO lj lq oj oq ql qo sb sy ud uw wf wu yh ys
2(60)2 BO bq bu bw dl dq ds dy fj FO fs fy hj hq hu hw jf jh jw jy ld lh lu ly
ob OF OS OW qb qd qs qu sd sf SO sq ub uh ul uq wb wh wj WO yd yf yf yl
2(0)2 BY dw fu hs jq LO ol qj SH uf wd yb

2(135)1 BE bk DE dm fe fn HE hp jk jm lk ln OM OP qn qp sk sv um uv wn wv yv yp
2(90)1 bm bn dk dp fb fk hm hn je jv le lv OE ov qe qv sm sn uk UP wk wp ym yn
2(45)1 bp bv dn dv fm fv hk hv jn jp lm lp ok ON qk qm se sp ue UN WE wm YE yk

1(120)3 ea ec eg ei KA kc kr kt MA mg mr mx nc ni nt nz pg PI px pz va vc vg vi
1(60)3 ER ET EX ez kg ki kx kz mc MI mt mz NA ng nr nx PA pc pr pt va vc vg vi

1(135)2 eb ed EF EH kb kj kl ks md mj mo MU nf nl nq nw ph po pq py vs vu vw vy

ej EL eo eq kd kf ku kw mb mh ms MY nb nh ns ny pd pf pu pw vj vl vo vq
ES eu ew ey kh ko kq ky mf ml mq mw nd nj NO NU pb pj pl ps vb vd vf vh

ee kk mm nn pp vv
ek EM EN ep ke km kn kv ME mk mp mv ne nk np nv PE pm pn pv vk vm vn vp
ev ve mn nm kp pk

os Worms in the OSPD (5-9 letters) and Webster's (14-16 letters)

avions	boxiest	stogies	pressing	reissuing
bonzer	coniums	stymied	profiter	repulsing
bronze	conquer	surfing	ptomains	rerunning
dhutis	crozier	torchon	puccoons	simplexes
erenow	curving	umpteen	quirkier	strapping
expels	cyphers	unnoted	ravingly	synergies
fixers	dispute	upboils	refrozen	thermions
frypan	dusting	weapons	resupine	trappings
gulley	eelpout	whereto	rotenone	underpins
hornet	egoists	wigwams	semiosis	unstepped
howler	eonisms	yolkier	slipform	upcurling
jinxes	excitor	zonulae	somewise	woodenest
lumpen	eyecups		specious	xanthones
mayfly	ghettos	atomizes	sprucing	
nosier	hoister	boxiness	stoppled	
nudely	imbrown	bronzier	sulphids	autotoxication
oboist	impulse	browsing	swimmier	carcinopolypus
octroi	informs	brunizem	symbiote	compunctionary
panzer	kippers	bryonies	tampions	gastrophthisis
patios	milksop	cerotype	thornier	herniopuncture
patois	misplay	chronons	toilsome	metantimonious
pawpaw	moonlet	courting	topsides	newspaperwomen
permit	myiasis	cowpokes	uncomely	nonformulation
plashy	noncoms	deposits	uncouple	predisputation
plenum	osmotic	dystonia	utilidor	psychoanalyzer
potted	oviduct	elytroid	voidness	semibituminous
probit	parsnip	emulsion		snippersnapper
punted	passion	expenses	bongoists	spermatophytic
puling	pathway	foreknow	conodonts	sulphoselenium
runoff	payment	genitors	costuming	ultimogenitary
sannop	peepuls	hawthorn	exserting	ultimogeniture
senior	platoon	horsefly	fogfruits	
snowed	plunges	inoculum	forgotten	noncorporeality
somite	pounces	laywomen	gusseting	semiconspicuous
spoken	punkier	luxating	immunises	supercongestion
subfix	quirted	maintops	isochrons	trypanorhynchan
throne	rontgen	misjoins	isoprenes	
topees	sayings	mistouch	misruling	consanguineously
tangun	scherzo	nowheres	nephrisms	disingenuousness
welper	serving	outprice	nonplused	mispronouncement
	skipper	poetiser	ovulating	
tropin	spangly	popeless	porticoes	
viotron	spoiler	pratique	quietudes	