In the May 1983 *Word Ways*, Helen Motamen describes a word puzzle contest she won in which the object was to construct a five-by-five letter array containing a maximum number of three-letter, four-letter and five-letter words across and down. By placing further requirements on this contest, one can convert it into an interesting logological problem: the construction of a five-by-five word square in which are nested four four-by-four and nine three-by-three word squares. In fact, there are a whole host of problems to consider. The five-by-five word square may be either single (the five different horizontal words are duplicated vertically) or double (there are five different horizontal words, and five different vertical words, ten words in all). If the five-by-five square is single, the embedded squares on the diagonal are also, but the nested squares located above or below the diagonal may be double or defective (contain two identical words in a row). If the five-by-five square is double, there are many possibilities for the thirteen nested squares: each one may be single, double, defective (contains two identical words in a row, or in a column) or hybrid (one or more, but not all, horizontal words match vertical words). The ideal solution – naturally, the hardest one to achieve – is to have a double five-by-five word square containing thirteen nested double squares, each one different.

**D A M A N** The prize-winning Motamen array, given at the right, does not qualify as a five-by-five word square because it contains the repeated word ANANA. Her hand search of Webster’s Third Unabridged (the test dictionary) uncovered fewer than 600 fully-nested five-letter words (ones containing all four-letter and three-letter words, such as SPARK: SPAR, PARK, SPA, PAR, ARK), and she was unable to construct a legitimate word square from this stockpile. In view of the experimental support value of 455 calculated in “How Many Words Support a Square” in the May 1992 *Word Ways*, it appears that she was somewhat unlucky. Or, it is entirely possible that her stockpile did contain a legitimate five-by-five square, but, working by hand, she failed to discover it.

If one enlarges the stockpile, the problem becomes easy, at least for the single five-by-five word square. We used a database of 15,719 five-letter words, containing all the Webster’s Second words in the Air Force Dictionary, all the words in the Official Scrabble PLus some for testing then we found 1373 randomly-chosen words (sixty-six per cent of the total) that characterize the word in question. Of these, 37 represented a valid sequence of 13 words. The first five, four arrays, including the first two, sixth and seventh word squares, are shown below. The main nested details on p. 37.

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(B,C,D,F,G,L,S)
D A M A N
H E R E
B L A S E R E S T
H E D E R
B L A S E R E S T
H E D E R
R E S T
T H E R E
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Scrabble Players Dictionary (1978), and Chambers Words (1985), plus some from the Oxford English Dictionary and Webster's Third. Testing them against 8,072 four-letter and 1,707 three-letter words, we found 1,244 of them to be fully nested. From these, the computer constructed 12,696 single five-by-five and 5,932 double ones. Interestingly, the support calculated from this data suggests that 188 randomly-chosen words from the 1,244-word stockpile ought to be enough to construct, on the average, a single five-by-five square; the corresponding support for a double square is 522. Fully-nested words are peculiarly amenable to square construction, making Motamen's failure to find one even more inexplicable.

A single word square with all nested squares single is, of necessity, a progressive word square (one in which each word in turn discards the first letter of the preceding word and adds a new letter at the end). Progressive word squares can be succinctly characterized by a sequence of nine letters: the five letters of the word in the top row, followed by the terminal letters of the second through fifth words. This is the basis for the summary of the 147 single word squares with all nested single squares, listed below. In each letter array, every sequence of nine letters represents a square. Read the array from left to right; when additional rows are encountered, one may follow either one (thus, BRASEREST, BRASHEDER and BRASHETHE are all word squares). Any of the parenthesized letters or letter-groups may be appended to the start of a sequence (thus, BASHEREST and TRASHERES are both squares). The numerical indicator 37(13) says that there are 37 squares in the corresponding array, and the longest sequence of letters in the array is 13. Certain arrays intersect. The first five arrays share the word square HETHEREST; the first four arrays also share the squares SOTHERES and ASHERETHER; the first two arrays also share the word square BASHERET; the sixth and seventh arrays share AREASERER. Collectively, these word squares form four directed networks with no internal loops; the main network consists of all arrays but the final three. For details on directed networks, see "A Word String Network" in May 1991, and the three-part "Directed Word Chain Networks" in May and August 1991.

(B,C,D,F,G,L,M,P,TR,W)ASHEREST 37(13)
ST
(B,C,S)ABASERER 30(14)
HEDER
THEREST
RD
EST
K
BLASEREST
HEDER
REST
THEREST
What about nested squares in a double word square? The 5,932 double squares contain a wide variety of nested squares within them: single, double, hybrid, defective. None of the double five-by-five squares contains nine double three-by-three squares, but there are probably several hundred with eight double three-by-three squares, with the ninth either single or hybrid. It is hard to count how many there are because the fault can occur in many different places; however, by choosing to accept defects in this or that place and putting appropriate instructions into the computer program, one can find squares like the ones illustrated below. In the first two pairs, the hybrid three-by-three is part of a hybrid four-by-four; in the last square, there is one single three-by-three in addition to eight doubles, and the four-by-four squares are all double ones.

A N A S T Y B A S T A S T A W N S T A W N B L A D E
  O t h e r O t h e r W a s H E W a s H E R E v e r
  W h o R E W h o R E a r e a R a r e a R A w e n e
  N e w E N N e w E N D e e E S E D e e E S E N e r e s
  E r e s T e r e s T S a l e S M a l e S T R A S T

What are the prospects for extending this study to nested six-by-six word squares? It seems unlikely that any can be found.
within the confines of Webster's Second, Webster's Third and the Oxford English Dictionary. If one tests the 1244 fully-nested five-letter words for overlapping crashes (that is, form letter-sequence ABCDEF from fully-nested words ABCDE and BCDEF) and determines which of these are in fact six-letter words, one finds 230 of them, listed below. These form the raw material for a six-by-six nested square, but, judging from the support statistics calculated for five-by-five nested squares, it is clear that a considerably larger set is needed.

abaser  brathe  croose  grasped  mother  rooses  souses  tether
abashe  bushe  dashed  grapes  muses  rowes  rowers  swahe  thawed
agates  cagers  dasher  grates  mowers  sahest  spales  thaver
agenes  carene  dashes  greese  musers  sables  spanes  themes
amates  carere  earest  grasped  nather  scaped  spared  theres
amides  carest  eather  gropes  neared  scaper  saper  thouse
amused  cashed  eraser  groups  nevels  scapes  spares  totems
amuser  casher  fashed  grouts  nooses  saced  spates  tawers
amuses  cashes  fasher  hairled  paired  scarer  staker  traped
ananas  catell  fashes  hewers  palays  scares  stakes  trapes
arears  chains  ferest  laches  parent  seathe  stapes  vowers
aseth  chaire  flaked  lament  nashed  serest  stared  wagers
atokes  chapes  flaker  lashed  pashed  sethen  starer  warent
aweels  shared  flakes  lasher  peares  sewers  stares  warest
baches  chares  flamen  lashes  please  shales  strade  washed
barest  chaved  flamer  lather  powres  shapes  stramp  washer
bashed  chaver  flanes  leared  prosed  shared  strays  washes
basher  chides  flemes  leares  proses  shares  swaged  wearad
bashes  chore  ferest  laches  parent  seathe  stapes  vowers
bather  choses  gagers  levels  raches  sheder  swages  wether
blasty  chouse  galays  lewest  ragers  shewel  taches  whares
booses  chout  gamene  looses  rasher  shewer  tavers  wheels
bother  clee  garest  lowers  rashes  shores  taver  wheres
bowers  clevs  gashed  mased  rather  shouts  teared  wholes
braked  cleath  gasher  masher  reaped  shower  teaser  whoops
braker  copen  gashes  mashes  revels  shules  teache  whores
brases  coweres  godsos  merest  revere  skates  tepees  wother
brasen  croons  gooses  mether  revert  sorest