A REMARKABLE REVELATION?

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Set $A=1, B=2, \ldots$ to create chains of number names: FOUR ($6+15+21+18$) has a score of 60, SIXTY has a score of 97, NINETY-SEVEN has a score of 152, and so on. In the 1960s, Howard Bergerson discovered that every number name, from ONE to NINE HUNDRED NINETY-NINE VIGINTILLION, \ldots, NINETY-NINE, homes in on one or another of the five numbers in the cycle \(-216-228-288-255-240-\). In Problem 128 (The Pentagon) in Beyond Language (Scribner's, 1967), Dmitri Borgmann described this in his extravagant style as possibly "the most remarkable, the most historic revelation about words in the entire evolution of human thought". It's hardly that - but on the other hand, the topic does not deserve the neglect it has suffered for nearly 30 years.

Although it is obviously impossible to diagram how a thousand vigin­tillion number names converge to the pentagon, one can give some sense of its general structure. Number names smaller than 80 always lead to larger number names; conversely, ones larger than 279 lead to smaller ones. Between 80 and 279 the behavior is more irregular; number names can be followed by either larger or smaller ones, and the chain sometimes wanders briefly outside these limits (80 leads to 74 leads to 170, 277 leads to 307 leads to 195). The longest possible chain from any number name to any number in the pentagon appears to be 17, achieved both by number names coming up from below (FOUR, as illustrated below) and for number names coming down from above. The latter descend to a manageable range very quickly indeed. Ed Wolpow in the August 1981 Word Ways noted that the most letters in a number name is 758; this corres­ponds to a score of approximately 9000. In turn, a number of this size has a score of typically 400, and the third step brings it down to 309 or less.

The five entry-points to the pentagon are unequally represented if one considers chains starting with number names of 1000 or less. 228 has no such chains, 255 has 14 chains, 288 has 15 chains, 216 has 28 chains, and 240 has all the rest of the 994 possible ones (excluding the five number names in the pentagon). Aside from 216 (in the pentagon), the smallest numbers leading directly to 228 appear to be 4050, 5040, 6080, 9040, 10012 and 11011. Does this unequal distribution persist for larger number names? This could be gauged by programming a computer to select very large number names at random and calculating the chains.

Some appreciation for the apparent fecundity of 240 can be gained by diagramming the tree of number names in the range 180 to 307 that converge upon it. The asterisks at the right identify most of those
chains that can be 17 steps long: the 17-link chain starting with FOUR
is also included.

+215
| +229-260-197-207
+238 +213
| +231-188-302-274
| +203
| +236-214 +244-198-250-241-217
| +275-293 +267 +287-262-283 +257-166-201*
| +297-237-195-190---304-217 +289 +295-223*
| +222 +211 +202 +171-210
| +190-304-287 +307-277-292-234-186-303 +152-97-60-4
+259 +245 +226 +209 +212 +230*
| +182-180 +286 +192 +268-298-273-221
| +306 +249 +246

One should note that any word, not just a number name, can be
classified according to entry points. Words of score 119 converge to
255; words of score 3,26,41,56,62,78,83,118,130,151,155,159,194 and 208
converge to 216. Would anyone like to write an essay corresponding to
216-convergent words?

How different might this mathematical structure look if the alphabet
were rearranged? What are the theoretical limits to which it can be
pushed? One answer to these questions was provided in the February 1990
Word Ways which showed that for the rearrangement .ESIV.F.WR.Y.UD..H.
TXOL.G.N one could create a structure in which all number names converged
to one or another of 38 cycles of length one (that is, to 38 self-referen-
tial number names) between 50 and 299. What alphabetic rearrangement
produces the longest possible cycle? How long might this be? What rearr-
arrangement produces the longest possible chain between a number name and
its entry into a cycle?

More subtle properties of the mathematical structure can also be
looked at. In most rearrangements having two or more independent cycles
to which number names converge, every cycle will have a very large

region of inf
However, it is
small set of
starting ISX+
ential number
large can suc
the region of

ENVIWT...L
leads to 9, a
augment this
4, 8 and 3 al
.H.WRNX...E1
1. 3, 41, 51
sets U=13 and
It is possibl
only a small
computer.
region of influence involving perhaps vigintillions of number names. However, it is possible to imagine cycles which attract only a very small set of number names to them. The simplest example is an alphabet starting ISX (or any permutation of these letters); SIX is a self-referential number, and no other number names can reach SIX by a chain. How large can such a structure be, either with respect to the cycle size or the region of its influence?

ENVIWT.O...LS leads to the self-referential number names 9 and 19; 10 leads to 9, and 11, 7 and 2 to 19; in turn, 1 leads to 11. One can augment this alphabet to ENVIWTUO..LSFY....GH.R.X.. which results in 16, 4, 8 and 3 all converging to the self-referential 50. TYOFGS.VI......H.WRXN.....E leads to the cycle -30-50-20-70-90-80-60-40- into which only 1, 3, 41, 51 and 61 feed (to 50, 90, 80, 70 and 90 respectively). If one sets U=13 and L=10, then 4, 12, 34, 44, 54 and 64 also enter the cycle. It is possible that a slightly larger cycle can be found that attracts only a small set of number names; this is left as an investigation for a computer.