A NEW KIND OF TRANSPOSAL SQUARE

A. ROSS ECKLER
Morristown, New Jersey

In my article "Single and Double Transposal Squares" in the May 1980 Word Ways, I generalized the concept of word square, allowing one to rearrange the letters in each row and column to form a word. Double transposal squares of size three are ridiculously easy to construct. The one on the left below is perhaps the commonest one, consisting of six words each having more than a thousand occurrences in Kucera and Francis's Computational Analysis of Present-Day American English (1967): man 1207, him 2619, its 1858, was 9816, who 2252, not 4609.

\[
\begin{array}{ccc}
M & I & H \\
A & S & W \\
N & T & O \\
\end{array}
\quad \begin{array}{ccc}
A & I & O \\
H & N & S \\
T & W & M \\
\end{array}
\]

The letters of a double transposal square of size 3 can be rearranged to form an array with a different set of properties. In the 3-by-3 grid shown at the right, the same six words can be read off in a different way: the letters of each word appear once in each row and once in each column.

This arrangement can be generalized to larger arrays. For instance, in a 4-by-4 array, there are 24 different ways in which one can select sets of four letters, one from each row and one from each column. Can an array be constructed in which all 24 letter sets are rearrangeable to words? This can be done; the words from the array below can all be found in the second edition of the unabridged Merriam-Webster:

\[
\begin{array}{cccc}
Y & H & B & K \\
E & S & N & L \\
I & A & R & T \\
U & O & C & D \\
\end{array}
\quad \begin{array}{cccc}
cyst & hunt & hind & etch \\
drys & rusk & sick & herd \\
lory & balu & boil & kore \\
lacy & kuan & kino & bade \\
Andy & hurl & bids & cake \\
tony & bust & lich & bote \\
\end{array}
\]

It is obviously impossible to construct a 5-by-5 array with 120 different words out of an array of 25 different letters. To have any hope of success, one must allow letter repetition. But here one must impose an additional restriction in order to avoid trivialities such as arrays consisting of the same 5-letter word repeated five times. The restriction is that no two of the 120 5-letter sets drawn from the rows and columns can be identical. It is a difficult mathematical problem to ascertain all the conditions guaranteeing that duplicates cannot be formed. (One such condition, for example, is that one cannot simultane-
ously place two As in columns a and b of one row, and two Es in columns a and b of a second row.)

Mike Keith programmed a computer to evaluate 5-by-5 arrays, checking which of the 120 5-letter sets in an array could be rearranged to form Merriam-Webster words. Not only did his program check that all letter sets were different, but it also evaluated neighboring arrays (those nearly the same as the original) to see whether a larger number of Websterian words could be found. Using techniques to move to ever-better arrays (climbing toward the summit of a mathematical "hill"), he discovered that one can frequently find arrays with 110 to 115 Websterian words—but a complete Websterian solution appears elusive.

Here are a couple of his best solutions, with 114 and 115 Websterian words, respectively:

<table>
<thead>
<tr>
<th>E L U R L</th>
<th>R R R R L</th>
</tr>
</thead>
<tbody>
<tr>
<td>E N E L S</td>
<td>A C I M S</td>
</tr>
<tr>
<td>A R I T S</td>
<td>A G E T O</td>
</tr>
<tr>
<td>A M U S T</td>
<td>A N U S A</td>
</tr>
<tr>
<td>A B O D A</td>
<td>A B O D E</td>
</tr>
</tbody>
</table>

12345 anise scree 12354 tined cedar 12435 atune truce
12453 dunes duroc 12534 teton actor 12543 noses scroo
13245 erase girse 13254 deter argid 13425 ureal grume
12452 Druse drugs 13524 lerot morga 13542 roses gross
14235 AEEMT inert 14253 deems nidor 14325 melia EEMNR
14352 deism rends 14523 moles moron 14532 smote snort
15234 tebet ribat 15243 BEESS Boris 15324 blite amber
15342 bessi Serbs 15423 blues rumbo 15432 tubes burst
21345 aisle easer 21354 tiled ardea 21435 Aleut urate
21453 duels douar 21534 lotte aorta 21543 loses roosa
23145 urase gears 23154 trued garad 23415 urare argue
23451 lured dugal 23514 roter agora 23541 loser goals
24135 autem antre 24153 mused adorn 24315 aimer ranee
24351 limed laden 24513 mores aroon 24531 motel talon
25134 butte rabat 25143 buses boars 25314 tribe abear
25341 belis bales 25413 rebus burao 25431 bluet tubal
31245 asale arise 31254 dealt Daira 31425 alula AEMRU
31452 lauds sudra 31524 allot aroma 31542 lasso soars
32145 sauna cares 32154 daunt darac 32415 anura Eruca
32451 dunal ducal 32514 trona caroa 32541 loans coals
34125 amula namer 34152 adsum darns 34215 marae reina
34251 medal ladin 34512 roams arson 34521 molal monal
35124 tubal Abram 35142 ABSSU brass 35214 bater baria
35241 bales bails 35412 bursa bursa 35421 bulla album
41235 alate irate 41253 deals radio 41325 alila ameer
41352 dials reads 41523 salol AM00R 41532 altos roast
42135 AANTU trace 42153 Sudan cardo 42315 arian ACEER
42351 Ladin decal 42513 arson coroa 42531 talon octal
43125 aural gamer 43152 sudra drags 43215 arear regia
Can either word set be completed using words outside of Merriam-Webster? I suggest the following from the Times Index-Gazetteer and the Oxford English Dictionary:

AELTT attle, 'mining rubbish'
ANRSU arsun, var of arson 'saddle-bow'
AEEMT tea me, var of 'team' (v2)
BEESS Sebes, Rumania
ABSSU Bussa, Nigeria or Ethiopia
AANTU Nauta, Peru

EEMNR Merne Merna, Australia
AEMRU Col de Maure, France
AMOOR Moora, West Australia
ACEER Cerea, Italy
ACORU Coura, Portugal

In the January 1999 and April 1999 issues of Wordsworth, Ted Clarke discusses a similar problem entitled "Magic Word Squares". There he constructs a 6-by-6 array with all 26 letters of the alphabet represented in the 36 spaces. A complete set of 720 words is, of course, impossible to find; without spending much time on the problem, he located eleven 6-letter words in his array.