PALINDROMIC WORD SQUARES

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Introduction

A word square (n-square) consists of n rows, each containing a different n-letter word, so that the first column has the same word as the first row, and so on. A palindromic square is a word square in which the words are palindromes, with as many as possible being different. The investigation reported below was inspired by Zoran Radisavljevic’s article (W05-178).

Let us construct a 7-letter square using as many different letters as possible. We can write ABCD for the first word before the rest of the word (CBA) is forced by the palindromic property. This word becomes the first column (word square property). The second row must begin with B, and we can continue EFG, with the rest of the word (FEB) being forced:

```
  A B C D C B A
  B E F G F E B
  C F H I H F C
  D G I J I G D
  C F H I H F C
  B E F G F E B
  A B C D C B A
```

Generalising, we find these properties of palindromic word squares:

1. For n-letter words, the number of words in the square is \( \frac{n}{2} \) rounded up, at most. Thus there are just four different words in the above square.

2. The number of different letters is limited, to \( 4 + 3 + 2 + 1 = 10 \) in the case of the 7- and 8-squares, 15 in the case of 9- and 10-squares, and so on, the formula being \( \frac{1}{2} h(h+1) \), where \( h \) is the half length rounded up to the next integer: eg half of 7 rounded up is 4, and \( \frac{1}{2} 4(4 + 1) = 10 \). The number of letters in a square is \( n^2 \), so the ratio of different letters to all letters for even-length squares is \( \frac{1}{2} h(h+1) \) divided by \( n^2 \). As in this case \( h \) is exactly half of \( n \), the ratio is \( 1/8 + 1/(4n) \). This is at a minimum of 12.5% when \( n \) is huge (realistically say 15% when \( n = 10 \)), and at its largest (sensibly speaking) at 18.75% when \( n = 4 \). This is not a great difference: the great majority of letters in palindromic squares are repeats. As the following table shows, when we include the values for odd-length squares, the percentage of letters that can be different is still around 15-20:

<table>
<thead>
<tr>
<th>Squares</th>
<th>no. of letters</th>
<th>maximum no. of different letters</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-square</td>
<td>36</td>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>7-square</td>
<td>49</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>8-square</td>
<td>64</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>9-square</td>
<td>81</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>10-square</td>
<td>100</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>11-square</td>
<td>121</td>
<td>21</td>
<td>17</td>
</tr>
</tbody>
</table>

3. A palindromic word square consists of a square element, repeated with its inversion, reflection, and inversion/reflection. In the square above, the square element defined by its side ABCD is necessarily repeated four times. Because the above example has an odd size, the four elements share a row and a column. These facts appear to lead to an easy way of making large squares. To make a palindromic 8-square, for example, one merely has to make a 4-square of 4-letter sequences, each being the start of a 8-letter palindrome. This is reminiscent of the efforts to
make 10-squares in the 1920s by using tautonyms. It is, indeed, very quick — but quick merely to show that there is an insufficient number of such specialised words.

4. Further, the basic square element itself consists of a triangle (ABCD/EFG/HIJ), so the whole square consists of a triangle repeated eight times, though squares with an odd number of rows will have triangles which share some letters. Thus, a given letter may be forced to appear eight times. In fact, in the above triangle, B, C, and F each appear 8 times; A, D, E, G, H, and I appear four times each; and only J escapes replication. The case of the 8-square is simpler, as the basic square is repeated four times with no sharing, so all letters appear either four or eight times: A, E, H, and J appear four times, and the rest eight times.

5. The diagonals are identical, and palindromic

**Summary of Results**

<table>
<thead>
<tr>
<th>Word length</th>
<th>Not place names</th>
<th>Place names</th>
<th>Total words</th>
<th>Total squares</th>
<th>No. of squares with max # different letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>142</td>
<td>65</td>
<td>207</td>
<td>2523</td>
<td>2058</td>
</tr>
<tr>
<td>5</td>
<td>1486</td>
<td>690</td>
<td>2176</td>
<td>1994231</td>
<td>878231</td>
</tr>
<tr>
<td>6</td>
<td>471</td>
<td>174</td>
<td>645</td>
<td>87877</td>
<td>7640</td>
</tr>
<tr>
<td>7</td>
<td>660</td>
<td>218</td>
<td>878</td>
<td>186843</td>
<td>314</td>
</tr>
<tr>
<td>8</td>
<td>162</td>
<td>8</td>
<td>170</td>
<td>49</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>308</td>
<td>27</td>
<td>335</td>
<td>1057</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>67</td>
<td>2</td>
<td>69</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>2</td>
<td>52</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The table above divides palindromic words of a given length into those which certainly can be used as place names (even if they are also plain words), and those which are not likely to be place names; and shows the number of squares that can be made from each set, and the number that can be made from a combined set. It also shows the number of squares having the maximum number of different letters, as given in the first table in this article.

Among other things, the table shows that there are consistently far more words than place names; that squares are more difficult to find at length 4 (too few words) and at lengths 8 and above (too many words to combine); that I found no squares above length 9; and I found no “heterogrammatic palindromic squares” above length 7. The 7-square numbers are much larger than Radisavljevic reported. His article has no squares with the maximum number of letters.

Squares where the diagonals form a word are plentiful in the case of smaller squares, but do not exist above length 7. The total number of diagonal squares is broken down below into three components: those where the diagonal consists solely of the repeated letter A, those where the square has the maximum number of different letters, and others. There are no diagonal palindromic 8- or 9-squares.

**Table of Diagonal Squares**

<table>
<thead>
<tr>
<th>Word length</th>
<th>Number of squares</th>
<th>No. of diagonal squares with max letters</th>
<th>Diagonal squares with AAA... diags</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2523</td>
<td>406</td>
<td>415</td>
</tr>
<tr>
<td>5</td>
<td>1994231</td>
<td>44150</td>
<td>132882</td>
</tr>
<tr>
<td>6</td>
<td>87877</td>
<td>1</td>
<td>4029</td>
</tr>
<tr>
<td>7</td>
<td>186843</td>
<td>0</td>
<td>532</td>
</tr>
</tbody>
</table>

In all the following sections, the squares are made from a mixture of place names and ordinary words, though selection will favour non-place names. In selecting the best squares, words which occur as headwords in major dictionaries count as superior, but in any case expressions are
valued if they appear in everyday communications. Diagonal words are inset. To save space, only the top half of each square is shown, and sourcing has been mainly constrained to the OED and Palindromicon. Place names are from the NIMA database. W2 = Webster’s Second. O = OED. Pa = Palindromicon.

## Palindromic 4-squares

### Best 4-squares with maximum number of different letters (3)
All words are OED headwords except SCCS (= Source Code Control System, 1972, Marc Rochkind at Bell Labs; or Skunk Creek Computing Services, Boulder, Colorado. In common use.), ISSI (Pa), ALLA (W2). All are in the most-used 80,000 expressions.

<table>
<thead>
<tr>
<th>ABBA</th>
<th>ALLA</th>
<th>ANNA</th>
<th>ATTA</th>
<th>BEEB</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEEB</td>
<td>LEEL</td>
<td>NOON</td>
<td>TOOT</td>
<td>ELLE</td>
</tr>
<tr>
<td>BOOB</td>
<td>DEED</td>
<td>ISSI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OPPO</td>
<td>ELLE</td>
<td>EMME</td>
<td>ESSE</td>
<td></td>
</tr>
<tr>
<td>LEEL</td>
<td>NOON</td>
<td>OPPO</td>
<td>PEEP</td>
<td></td>
</tr>
<tr>
<td>EMME</td>
<td>OPPO</td>
<td>ESSE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>POOP</th>
<th>SEES</th>
<th>TOOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTTO</td>
<td>ELLE</td>
<td></td>
</tr>
</tbody>
</table>

### Best diagonal 4-squares with maximum number of different letters (3)

<table>
<thead>
<tr>
<th>ABBA</th>
<th>AEEA Pa</th>
<th>ANNA</th>
<th>ATTA</th>
<th>BEEB</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCCB</td>
<td>ABBA</td>
<td>BBBB</td>
<td>BAAB</td>
<td></td>
</tr>
<tr>
<td>BCCB</td>
<td>ABBA</td>
<td>BBBB</td>
<td>BAAB</td>
<td></td>
</tr>
</tbody>
</table>

### Best diagonal 4-squares regardless of number of letters

<table>
<thead>
<tr>
<th>AMMA</th>
<th>DDDD</th>
<th>ELLE</th>
<th>EMME</th>
<th>ESSE</th>
<th>LEEL</th>
<th>OPPO</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMMM</td>
<td>DEED</td>
<td>LEEL</td>
<td>MMMM</td>
<td>SCCS</td>
<td>or</td>
<td>SEES</td>
</tr>
<tr>
<td>OTTO</td>
<td>POOP</td>
<td>SEES</td>
<td>TOOT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOOT</td>
<td>OPPO</td>
<td>ESSE</td>
<td>OTTO</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In diagonal 4-squares, the diagonal may be the same as one of the rows, thus:

starting with:

<table>
<thead>
<tr>
<th>ABBA</th>
<th>if A=B</th>
<th>AAAA</th>
<th>if B=C</th>
<th>ABBA</th>
<th>if A=C</th>
<th>ABBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCCB</td>
<td>ABBA</td>
<td>BBBB</td>
<td>BAAB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCCB</td>
<td>ABBA</td>
<td>BBBB</td>
<td>BAAB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABBA</td>
<td>AAAA</td>
<td>ABBA</td>
<td>ABBA</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Best diagonal 4-squares regardless of number of letters

<table>
<thead>
<tr>
<th>AAAA Pa</th>
<th>AAAAA Pa</th>
<th>ABBA O</th>
<th>ATTA O</th>
<th>BBBB Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEEB O</td>
<td>ANNA O</td>
<td>BBBB Pa</td>
<td>TTTT*</td>
<td>BEEB O</td>
</tr>
<tr>
<td>ABBA O</td>
<td>ANNA O</td>
<td>ABBA O</td>
<td>ATTA O</td>
<td>BEEB O</td>
</tr>
</tbody>
</table>

*Teaching Technology Through Tradition - see Internet †also Mohammed Atta
*4D is a Canberra based company specialising in military spatial information systems.

**Palindromic 5-squares**

### Best 5-squares with maximum number of different letters (6) (all are OED headwords)

- **LEMEL**
- **LEMELE**
- **MAPAM**
- **NAMAN**
- **NEMEN**

- **EBIBE**
- **ERORE**
- **ALULA**
- **ALULA**
- **ERORE**

- **MINIM**
- **MODOM**
- **PUTUP (put-up)**
- **MUTUM**
- **MODOM**

- **NETEN**
- **RENER**
- **SENES**
- **TENET**

- **EBIBE**
- **EBIBE**
- **EBIBE**
- **EBIBE**

- **TIMIT** or **TIPIT**
- **NIKIN**
- **NIKIN**

- **VENEV**
- **WEREW** or **ZEREZ**

- **EBIBE**
- **EISTIE**

- **NIKIN**
- **RSFSR**

*or NISIN. RSFSR = Rossiiskaya Sovetskaya Federativnaya Sotsialisticheskaya Respúblika*

### Best 5-squares regardless of number of letters

The following words are all in the most used 90,000 English words.

- **KARAK**
- **MALAM**
- **SALAS**

- **ABABA**
- **ARARA**
- **W2**

- **RADOAR**
- **LAVAL**
- **LAVAL**

### Best diagonal 5-squares with maximum number of different letters (6):

- **MARAM**
- **MUTUM**
- **SEDES**
- **SELES**

- **AIEIA†**
- **UAIAU*†**
- **EBIBE**
- **EBIBE**

- **RENER O**
- **TIPIT O**
- **DIOID†**
- **LIOIL†**

- **MINIM O**
- **MAPAM O**
- **SBOBS O**
- **SBOBS O**

*Corrego Uaiau, Brazil, -16°45', -55°02' †Dioïd algebra, Lioïl Merrett - see WW 2003-114 ‡Association internationale des Ecoles et Instituts d’Administration (see Internet)*

### Best diagonal 5-squares regardless of number of letters

- **SERES**
- **STATS**
- **STATS**

- **EBIBE**
- **TENET**
- **TOROT**

- **RIOIR Pa**
- **ANDNA Pa**
- **ARKRA†**

- **SBOBS O**
- **SEDES O**
- **SOKOS O**

†Arkra Inc. - see WW 2003-114
Palindromic 6-squares

Best 6-squares with maximum number of different letters (6):

MARRAM O
ALEELA Pa or AREERA Pa
REDDER O or RENNER O

MODDOM O PULLUP O RATTAR O
OLEELO Pa UREERU Pa ABIIBA Pa or ADIIIDA Pa
DENNED O or DESSED O LEFFEL O TIPPIT O

RATTAR O RETTER O SIRRIS O or IMAAMI Pa or INAANI Pa
ALEELA Pa ESAASE Pa ILAALI Pa
TENNET O TALLAT O RATTAR O

TALLAT O WORROW O WORROW O
AREERA Pa ODAADO Pa OLEELO Pa
LEFFEL O RATTAR O REDDER O or RENNER O or REPPER O or RETTER

Best 6-squares regardless of number of letters

The best squares are the MARRAM squares above.

Best (only!) diagonal 6-square with maximum number of different letters (6)

SIDDIS Pa
IRAARI Pa
DAOOAD Darya-i-Daooad, Afghanistan, 37°17, 69°27
SROORS Pa

Best diagonal 6-squares regardless of number of letters

MAMMAM O MARRAM O REDDER O RETTER O
AAAAAA Pa AAAAAA Pa EEEEEEE Pa EEEEEEE Pa
MARRAM O RARRAR Pa or RATTAR O DEDDED Pa TENNET O
MARRAM O or MATTAM Pa REDDER O RENNER O
REDDER O RENNER O TERRET O
EEEEEEE Pa EEEEEEE Pa
DENNED O or DEPPED Pa NEDDEN Pa or NENNEN Pa RENNER O
RENNERO or REPPER O REDDER O or RENNER O TENNET O

RERRER Pa
EEEEEEE Pa
REDDER O or RENNER O
REDDER O or RENNER O
Palindromic 7-squares

Best 7-squares with maximum number of different letters (10):

DEVOVED O
ELIKILE Pa
VITATIV Pa
OKANAKO or OKARAKO or OKASAKO (all Pa)
MARGRAM Pa
ATENETA Pa
REVIVER O
GNIDING Pa or GNIPING Pa

Best 7-squares regardless of number of letters

CINENIC O
IBAAABI Pa
NAURUAN O
ANIHINA Pa or ANIKINA Pa
SARORAS Pa
AMEDEMA Pa or ATENETA Pa
REVIVER O
ODINIDO Pa or ONILINO Pa

Best diagonal 7-squares

SHAHAHS Pa
HADADA O
ADIAIDA Pa
HAALAAH Pa
SAILIAS Pa

SRETERS Pa
REPARER O
EPEREPE*
SARDRAS Pa
SEEDEES Pa

*1878 Browning, La Saisias

*Grove: Dictionary of Music: Octet, or Ottett (Ottetto)

and 5 squares from:

HADADA O
ACECECA or AGELEGA or AMEDEMA or AMELEMA or ATELETA, all Pa
DEVOVED O or DEDEEDEED O
ACOSOCA Pa or ALOKOLA Pa or ADDADDA Pa

*Rio Eperepe, Mozambique, -14°43, 38°46
Palindromic 8-squares

Best 8-squares

<table>
<thead>
<tr>
<th>Squares</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>DAIIQIQAD Pa</td>
<td>DEEDED Pa</td>
</tr>
<tr>
<td>AMASSAMA Pa or ANASSANA Pa or AVASSAVA Pa</td>
<td>ELARRALE G4p</td>
</tr>
<tr>
<td>QAMAAMAQ Pa</td>
<td>DAIIQIQAD Pa</td>
</tr>
<tr>
<td>ISATTASI Pa</td>
<td>ERICCIRE Pa</td>
</tr>
<tr>
<td>DOORROOD Pa</td>
<td>DREPPERD Pa</td>
</tr>
<tr>
<td>OOOOOOOO Pa</td>
<td>REDEEDER Pa</td>
</tr>
<tr>
<td>OOCOOCOO Pa or OOROOROO Pa</td>
<td>EDIEEIDE Pa</td>
</tr>
<tr>
<td>ROODDOOR Pa</td>
<td>PEEPPEEP Pa</td>
</tr>
<tr>
<td>ELARRALE*</td>
<td>LOOPPOOL Pa</td>
</tr>
<tr>
<td>LIVEEVIL Pa</td>
<td>OOOOOOOO Pa</td>
</tr>
<tr>
<td>AVATTAVA Pa</td>
<td>OOCOOCOO Pa or OOROOROO Pa</td>
</tr>
<tr>
<td>RETEETER Pa or RETOOTER Pa</td>
<td>POOPPOOP Pa</td>
</tr>
</tbody>
</table>

*Nor Arrale, Somalia, 2°58, 46°33*

<table>
<thead>
<tr>
<th>Squares</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NORAARON Pa</td>
<td>ROODDOOR Pa</td>
</tr>
<tr>
<td>OKERREKO Pa</td>
<td>OOOOOOOO Pa</td>
</tr>
<tr>
<td>REPOOPER Pa or RETOOTER Pa</td>
<td>OOCOOCOO Pa or OOROOROO Pa</td>
</tr>
<tr>
<td>AROLLORA Pa</td>
<td>DOORROOD Pa</td>
</tr>
<tr>
<td>OOROOROO Pa</td>
<td>SAASSAAS Pa</td>
</tr>
<tr>
<td>OKERREKO Pa</td>
<td>AJATTAJA Pa</td>
</tr>
<tr>
<td>REDEEDER or REPEEPER or RETEETER (all Pa)</td>
<td>AAAAAAAA Pa</td>
</tr>
<tr>
<td>OREMNERO Pa or OREMNERO Pa</td>
<td>STANNATS Pa or STARRATS Pa</td>
</tr>
<tr>
<td>SLEFFELS Pa</td>
<td>SNIFFINS Pa</td>
</tr>
<tr>
<td>LEERREEL Pa</td>
<td>NEILLIEN Pa or NOILLION Pa</td>
</tr>
<tr>
<td>EEEEEEEE Pa or EEEELEE Pa</td>
<td>IITOOTII Pa</td>
</tr>
<tr>
<td>FREDDERF Pa</td>
<td>FLOWWOLF Pa</td>
</tr>
</tbody>
</table>

Palindromic 9-squares

Best 9-squares

5 squares from:

<table>
<thead>
<tr>
<th>Squares</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ASSAMASSA Pa</td>
<td>SAMASAMAS Pa</td>
</tr>
<tr>
<td>SHALALAIHS Pa or SHANANAHS Pa or SHARARAHAS Pa</td>
<td>ALANANALA Pa</td>
</tr>
<tr>
<td>SARALARAHS Pa</td>
<td>MALAYALAM O</td>
</tr>
<tr>
<td>ALANANALA or ANANANANA or ANASASANA or ARAKAKARA or ARARARARA all Pa</td>
<td>ALANANALA Pa</td>
</tr>
<tr>
<td>MALAYALAM O</td>
<td>MALAYALAM O</td>
</tr>
<tr>
<td>AKAHAKAHA PA</td>
<td>ANANANANA Pa or ANASASANA Pa</td>
</tr>
<tr>
<td>SAYAPAYAS Pa or SAYASAYAS Pa</td>
<td>SAYAPAYAS Pa or SAYASAYAS Pa</td>
</tr>
</tbody>
</table>
SAMASAMAS Pa
AMAMAMAMA Pa
MALAYALAM O
AMARARAMA Pa
SAYAPAYAS Pa or SAYASAYAS Pa
SAMASAMAS Pa
ANANANANA Pa
MALAYALAM O
ANASASANA Pa
SAYAPAYAS Pa or SAYASAYAS Pa
SARALARAS Pa
ANONANONA Pa
ROTAVATOR O
ANANANANA Pa or ANASASANA Pa
LAVALAVAL Pa or SAVASAVAS Pa
4 squares from:

SAMASAMAS Pa
AMARARAMA Pa
MALAYALAM Pa
ARAKAKARA Pa or ARARARARA Pa
SAYAPAYAS Pa or SAYASAYAS Pa
SAMASAMAS Pa
ARARARARA Pa
MALAYALAM O
ARAKAKARA Pa
SAYAPAYAS Pa or SAYASAYAS Pa
SARALARAS Pa
ARORARORA Pa
ROTAVATOR O
ARAKAKARA Pa or ARARARARA Pa
LAVALAVAL Pa or SAVASAVAS Pa
SAMASAMAS Pa
ARAKAKARA Pa
MALAYALAM O
AKAHAAHAKA Pa or AKAKAKAKA Pa
SAYAPAYAS Pa or SAYASAYAS Pa

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