

## USING MATHEMATICAL PARENTHESES IN ENGLISH

Solomon W. Golomb  
Los Angeles, California

Parentheses are used in mathematics to indicate that certain computations should precede other computations, and also to shorten expressions by virtue of the “distributive law” that says  $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$ . Both of these uses can be applied to English text. A classic chestnut writes

**((N + H)OW + (T + W)HAT)(I KNOW)**

as shorthand for the introspective, epistemological statement:

**NOW I KNOW HOW I KNOW THAT I KNOW WHAT I KNOW,**

which expands the original 14 letters to 41 letters.

Smaller examples of this type of shorthand include: **(D + T)OWN** for **DOWNTOWN**; **(T & G)OWN** for **TOWN & GOWN**; **(N + L)IGHT** for **NIGHTLIGHT**; **(F + W)INE** for **FINE WINE**; etc. The reader is encouraged to find additional examples, and to look for phrases involving three or more “rhyming” words. (We only care about “eye rhymes”, the kind where **MATURE**, **NATURE**, and **STATURE** are “rhyming” words.)

Most operations of ordinary arithmetic are *associative*:  $a + (b + c) = (a + b) + c$  and  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ . However, in general,  $a^{(b^c)} \neq (a^b)^c$ . For example,  $(2^3)^2 = 8^2 = 64$ , but  $2^{(3^2)} = 2^9 = 512$ . The number of possible ways to insert parentheses grows exponentially with the number of letters in the exponential tower, following the sequence 1, 1, 2, 5, 14, 43, 132, ... of the “Catalan numbers”. This is mirrored by English phrases, which can have multiple meanings depending on where we put the parentheses, which are rarely written, but must be inferred.

For example, “A fine gold ring”, if thought of as “A fine (gold ring)” means a gold ring that is fine, but “(A fine gold) ring” means a ring made of fine gold. With three words (ignoring the article “a”), the Catalan number **2** tells us we have two essentially different ways to insert parentheses. The next Catalan number is **5**. Remarkably, a phrase with four words (ignoring articles) can have five different meanings depending on where we put the parentheses. Such a phrase is

### **A PRETTY LITTLE GIRLS SCHOOL**

Compare  $((2^2)^3)^2 = ((4)^3)^2 = 64^2 = 4096$ ;  $(2^2)^{(3^2)} = 4^9 = 262,144$ ;  
 $(2^{(2^3)})^2 = (2^8)^2 = 65,536$ ;  $2^{((2^3)^2)} = 2^{(8^2)} = 2^{64} = 18,446,744,074,029,551,616$ ;  
 $2^{(2^{(3^2)})} = 2^{(2^9)} = 2^{512} = 1.34078... \times 10^{154}$ , with five very different numerical values.

Here is our corresponding verbal situation:

- 1) A PRETTY (LITTLE (GIRLS SCHOOL)),  
A girls' school that is little and also pretty.
- 2) A (PRETTY LITTLE) (GIRLS SCHOOL),  
A girls' school that's *pretty little* (i.e., fairly small).
- 3) A PRETTY ((LITTLE GIRLS) SCHOOL),  
A pretty school for little girls.
- 4) A (PRETTY (LITTLE GIRLS)) SCHOOL,  
A school for little girls who are pretty.
- 5) A ((PRETTY LITTLE) GIRLS) SCHOOL,  
A school for girls who are fairly small (i.e. pretty little).

A challenge for the readers of WORD WAYS:

Find a phrase with five words (not counting articles) that can be interpreted in *fourteen* different ways, depending on where and how the parentheses are inserted.

When we deal with palindromes (are they invented, or merely discovered?), we usually don't care about word spaces or internal punctuation marks. Thus, on the model of **ABLE WAS IERE I SAW ELBA**, my example **AN OLE CRAB WAS IERE I SAW BARCELONA**, and the classic **MADAM I'M A DAM**, are considered perfectly acceptable. With this in mind, but forgetting palindromes, what do you make of the following message:

“To Beorn, Otto, Beth,  
At Isthèque St.-Ion”?