A PERFECT WORD

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Perfect numbers (PNs) are integers whose factors sum to the number, like 6 (=1+2+3). Euclid gave a formula for finding them: if $2^n - 1$ is prime then $2^{n-1}(2^n - 1)$ is a PN. There are only five PNs below ten billion: 6, 28, 496, 8128 and 33,550,536.

By analogy a perfect word is defined by using alphanumerical values of its letters as the factors (A=1 to Z=26, wrapping mod26 for factors above 26). If the values sum to their lowest common numerator they make a PN and the word is perfect. In pattern words some letters occur more than once. These can be treated in two ways here: the duplicated letters can be counted either as one or as multiple factors. In perfect numbers, factors that divide into the number more than once are only counted once. But first let’s consider the case in which every letter contributes a factor, so limiting the search to heterograms.

There is only one perfect heterogram, cab, plus its inferior anagrams, ABC and bac.* To justify this claim, consider the other PNs. 28 (1+2+4+7+14) yields the letters ABGDN for which I found no transposal except a conceivable poetic nonce bang’d. 496 (1+2+4+8+16+31+62+124+248) yields ABDDHENJT with no transposal word and but two poor phrases, an anti-marijuana slogan, “Bend J path, pend J bath!” It gets worse. For 8128 (1+2+4+8+16+32+64+128+256+508+1016+2032+4064) the letters are useless: ABDDHPFLWNTBDH. Ditto for 33,550,336’s ABDDHPFLXVRJTNABDHPFLX VRJT. Euclid’s formula means PNs are based on two series of progressive doublings, so only two factors will be odd numbers. Since all seven vowels are odd-numbered letters, larger PNs’ letters will be virtually vowelless, hence wordless. QED. (Unless a word can be found for 28 or 496.)

* Abc (usually plural) is a ‘word’ in Web-3 but not in Official Scrabble Words (OSW) or Official Scrabble Players Dictionary. Bac, short for baccalaureate, is a word in OSW only but is in Web-3 as an abbreviation for bachelor.

When we consider the more appropriate case of counting repeated letters only once, several more words are perfect, all for PN 6: abac, abaca, bacaba, bacca, Caaba and caab. (All are in Web-3 except abac, from OSW.) However, none of these is a common word in the West, although Caaba (or Kaaba, the sacred shrine in Mecca) is common in Islamic countries, of course. (It should please Muslims that Caaba is a perfect word.) I found no words from the alphomes ABDDGN (28) or ABDEHJLPT (496). Can readers find any? If not, cab is by either criterion the only common, perfectly good perfect word.

So hold that cab! Yea, embrace and cherish it. This pedestrian (well, pedestrian-serving) item is uniquely interesting and so deserves a little attention. As well as taxi, Web-3 gives four other definitions for cab: an ancient Hebrew measure of about two quarts; Scot slang for purloin; dialect for something sticky or dirty; and short for caco-chon, a form, style or cut of gems. Cassell Dictionary of Slang adds three more uses: a Cavalier (<Sp. Caballero>); a brothel; and a students’ cheat crib (short for cabbage). As to cab the vehicle, Chambers Dictionary of Etymology cites its first appearance as 1826 in the early Disraeli novel Vivien Grey as a short form of cabriolet, a horse-drawn carriage, itself borrowed by Smollett in 1763 from French cabriolet, from cabrioler, caper or leap (because of its bouncy motion), ultimately deriving from the Latin for wild goat, capreolus. This is only fitting since over the past century or so cabs have been involved in many tragedies (goat songs).