

AFFINE WORD GEOMETRIES

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Recall that a finite Affine plane of order n must have n^2 points and n^2+n lines, with n points to a line. There must be at least three non-collinear points and the plane must satisfy the axioms:

A1: Any two points determine a unique line.

A2: Given any line and a point not on that line there is a unique line through the point parallel to the given line (i.e. the lines have no point in common).

Ordinary Euclidean geometry is an example of an infinite Affine geometry since the two axioms are valid in the plane. In this article I will construct finite Affine geometries of orders 2,3, and 4. My good friend Norwich Bumstead offers an order 5 geometry in the article that follows. There is no order 6 geometry and higher orders seem beyond realization with English words.

The SIAM order 2 Affine geometry uses the square

| | |
|----------|----------|
| S | I |
| A | M |

The “points” are the $2^2 = 4$ letters of SIAM and the $2^2 + 2 = 6$ “lines” are the words: IS, AM, (row rook sweeps of the square), AS, MI (column rook sweeps) and MS, AI (bishop sweeps of the diagonals). All six word-lines are familiar main entries in most dictionaries. The reader can easily verify that the axioms are met and this is an order 2 Affine geometry.

For the order 3 geometry we use the letters of ADONIS to form the $3^2 = 9$ “points” listed in this 3x3 square.

| | | |
|-----------|-----------|-----------|
| AD | ON | IS |
| IN | AS | DO |
| SO | ID | AN |



For our $3^2 + 3 = 12$ “lines” we form 3 rook sweeps of the rows AD-ON-IS, IN-AS-DO, and SO-ID-AN; and 3 rook sweeps of the columns AD-IN-SO, ON-AS-ID and IS-DO-AN. The other 6 “lines” are bishop sweeps on the letters of ADONIS, namely AD-AS-AN, AD-DO-ID, ON-DO-SO, ON-IN-AN, IS-ID-IN, IS-AS-SO. These are all the diagonals of the square including the broken ones.

It is clear that any two points are on a unique line here (either on a rook sweep or a bishop sweep). And there are four sets of three parallel lines; two rook sets and two bishop sets. These can be used to verify axiom 2. Notice also that our 3x3 square is a semimagic word square, that is, one whose three entries in any row or column anagram to the magic constant ADONIS. See the references for more examples of such squares.

The order 4 plane employs the letters DOUBLES MATCH. The $4^2 = 16$ “points” are the 16 words in the following square.

| | | | |
|------------|------------|------------|------------|
| SHO | MET | LUD | CAB |
| BED | COL | HAT | SUM |
| CUT | SAD | MOB | HEL |
| LAM | HUB | SEC | DOT |

This square is semimagic with magic constant DOUBLES MATCH on the rows and columns. Most of its entries are familiar words and can be found in, say, the OED. The less familiar SHO, A Japanese mouth organ and LUD, a lord also appear there and HEL, the Scandinavian goddess of the dead is an entry in the Encyclopedia Britannica (11 ed.).

The chess piece motif continues to locate the $4^2 + 4 = 20$ “lines” of this geometry:

4 Rook sweeps of the rows. E.g. LAM-HUB-SEC-DOT.

4 Rook sweeps of the columns. E.g. CAB-SUM-HEL-DOT.

4 Bishop sweeps on vowel diagonals. E.g. MET-BED-SEC-HEL is the E-Bishop tour.

8 Knight 4-tours on consonants. E.g. SHO-SAD-SUM-SEC is the S-Knight tour.

It is, once again, straight forward to verify the axioms for these points and lines.

Several games can be played on these geometries and will be the subject of a future article in *Word Ways*. Meanwhile an interesting puzzle can be had by asking someone to form the 4x4 square whose rows and columns form DOUBLES MATCH and whose main diagonals each contain words of a unique vowel. There are several solutions but they are not easy to find without noticing the chess piece scheme.

References:

- (1) Farrell, J. "Magic Square Magic", *Word Ways*, 33(2), May, 2000, pp. 83-92.
- (2) Kim, S. "Mind Games", *Discover Magazine*, Nov. 2006, p. 74.
- (3) Pickover, C. A. *The Zen of Magic Squares, Circles and Stars*. Princeton Univ. Press, 2002. pp. 14-146.
- (4) Thumpbindle, O. "Most-Perfect Word Magic", *Word Ways*, 40(4), Nov. 2007, pp. 293-296.
- (5) Thumpbindle, O. "The Stenographic Affine Plane", Handout, Gathering 4 Gardner 8, March, 2008.

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