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by

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Abstract

Textbooks in Environmental and Natural Resource Economics invariably deal with the problem of allocating a non-renewable resource over time. The simplest version of that problem is the case of a resource that is to be allocated over two periods. The resource has a constant Marginal Extraction Cost (MEC). Most textbooks treat this case before moving on to more complex and realistic cases. This paper suggests the results that should be emphasized and the method that should be used to arrive at those results. It also points out the possible confusions that should be avoided. Finally, it examines how several well-known textbooks treat this issue.

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The efficient allocation of a non-renewable resource over time and the performance of competitive markets in allocating such a resource are fundamental to understanding the general role of markets in allocating natural resources and the concept of sustainability. In most textbooks, the first look at this issue comes in the form of a simple example showing the efficient allocation of a non-renewable resource over two periods. The demand for the resource is assumed to be the same in the two periods, the MEC is the same for all units, and the interest rate is assumed to be positive.

The example should derive the efficient time path of consumption by showing that it maximizes the sum of the present discounted values of consumption benefits minus extraction costs in the two periods. If possible, it should do so without resorting to calculus, because many students in these courses have limited mathematics background. The example should illustrate the efficient time path of price and quantity using the concept of user cost (the notion that the amount of net benefit lost in period 1, if one additional unit is consumed in period 0, is the difference between the marginal benefit in period 1 and the MEC, adjusted for the time value of money). Maximization of the PV of net consumer benefits requires that $(MB_0 (Q_0) – MEC) = (MB_1(Q_1) – MEC)/(1 + r)$, where $Q_0 + Q_1 = Q_T$, the total amount of the resource available.

Having derived the efficient time path, the discussion should then show that a competitive market (containing many independent resource owners possessing knowledge of the present and future extraction costs and demands) would arrive at the same outcome. An easy way to obtain this result is to show that a wealth-maximizing owner would decide when to extract and sell by comparing (P_0– MEC) to (P_1 – MEC)/(1+r). If the former is greater than the latter, it would sell in period 0. If the opposite is true, it would sell in period 1. If they are equal, the firm would be indifferent as to when to extract and sell. Since all sellers would want to sell in the period with the higher PV of (P – MEC), the only allocation that would result in some of the good being sold in each period is the allocation at which P_0 – MEC = (P_1 – MEC)/(1 + r). This is essentially an arbitrage story. Rational behavior by consumers guarantees that P_i = MB_i, so the competitive market allocation is identical to the socially efficient allocation. It may be useful to note in passing that the competitive equilibrium obeys the "Hotelling Rule," which requires the difference between price and extraction cost to increase at the rate of interest over time in cases such as this one. The goal of the analysis should be to explain in an accessible way (without calculus) what the efficient time paths of price and consumption look like and to show that a well-functioning competitive market duplicates this efficient outcome.

Having shown these things, one can move on to more complex stories, such as "backstop" technologies and non-constant marginal extraction costs. This paper does not concern itself with these extensions. Rather, I want to report and comment on how the leading texts dealing with natural resource economics issues handle the simple two-period constant MEC case.

How Do the Leading Texts Handle This Problem?

It is perhaps surprising to learn that most of the texts I examined fall short of the ideal presentation of this concept in one or more ways. In one case, the presentation contains so many errors that it fails in all the key areas.

One of the best treatments is found in a book on energy economics (Griffin and Steele, 1984). Griffin and Steele begin by focusing on a firm that owns a non-renewable resource and working out the implications of wealth maximization. In the course of their discussion (pp. 68-78), the authors explain the notion of user cost (or, as they prefer to call it, user value) without specifying what sort of firm is being discussed or how marginal revenues at various future dates are determined. In that general setting, Griffin and Steele derive the rule that, given a set of expectations about future costs and demands, user cost (MR_t –MEC_t) must rise at the rate of interest. They also show that changes in expectations about either future demand or extraction costs can cause the time path of user costs to change. Having thoroughly discussed user cost, they move on to Hotelling's result, which, as they point out, is based on the assumption of a competitive market. They do an excellent job of showing (in the two-period case) how the equilibrium reached in a competitive market consisting of many wealth-maximizing resource owners would result in the user cost's rise at the rate of interest. This presentation meets almost all the standards of a good presentation of this issue. It gives a thorough introduction to user cost, shows that user cost must rise at the rate of interest for a resource owner who plans to sell some of the resource in all periods, derives the Hotelling result, and shows convincingly why a competitive market in equilibrium must display the Hotelling result. In the course of showing the Hotelling result for a competitive market, they introduce a diagram that finds its way, in one form or another, into many later textbooks. This shows the derivation of the $(P_0 - MEC) = (P_1 - MEC)/(1+r)$ condition by measuring Q_0 from left to right and Q_1 from right to left. $P_1 - MEC$ is discounted, and the width of the diagram is $Q_T = Q_0 + Q_1$. The numerical example that lies behind the diagram is as follows: Demand in each period is given by $P_t = 50 - 0.5 \cdot Q_t$ ($t = 0,1$). MEC = 0. The interest rate is $r = 0.1$. Their diagram is reproduced here as Fig. 1.

[Insert Fig. 1 here.]

Griffin and Steele tell their entire story without calculus. Only two important and related concepts are missing: a derivation of the *socially efficient* allocation of a non-renewable resource and the demonstration that the competitive equilibrium allocation is socially efficient.

Tom Tietenberg's (2003) text treats this issue in three places and contains no serious errors. However, he does rely on calculus to tell the basic intertemporal optimization story (pp. 33 and 101), and his discussion of the market allocation of the resource, focusing on the efficiency of the outcome, does not use wealth-maximization and arbitrage behavior by resource owners to show how the market equilibrium is arrived at. His discussion is couched entirely in terms of what an *efficient market* would do: "An efficient market would have to consider not only the marginal cost of extraction for this resource, but the marginal user cost as well. Whereas in the absence of scarcity, the price would equal the marginal cost of extraction; with scarcity, the price would equal the sum of the marginal extraction cost and marginal user cost" (p. 92). While Tietenberg correctly states the conditions that a competitive market would have to meet in order to be efficient, he does not show how wealth-maximizing behavior of competitive resource owners moves the market to the efficient outcome. The next authors considered do a much better job on that score.

Hartwick and Olewiler (1998) discuss this case at some length (pp. 274-283). They solve the problem using the method of Lagrange. As mentioned above, it would be nice to tell the story without using calculus. They commit one error in the description of their Lagrangian problem. The maximand is stated as $[B(q_0) – C(q_0)] + [B(q_1) – C(q_1)]/(1+r)$. They go on to say, "$B(t_i)$ is the *consumer surplus* from the extracted mineral in period t (t = 0, 1)." (Emphasis added.) In fact, in this problem, B should represent total benefit to consumers (the area under the demand curve), not consumer surplus. This is the only error in an otherwise excellent treatment. They obtain the "Hotelling Rule" directly as part of the solution to the intertemporal efficiency problem. They then show that, if prices followed that pattern, all owners would be indifferent as to when to extract.

Another very good (though brief) treatment is by Eban Goodstein (2008, Appendix 6A, pp. 112-115). Goodstein manages, without using calculus, to show how a competitive market would allocate a non-renewable resource over two periods. He gives a correct statement of the Hotelling result and (in a footnote) sketches why the competitive outcome is efficient. I am not sure the discussion is long enough to give students a chance to understand it fully, but it is an impressive piece of concise writing.

A new book by Keohane and Olmstead (2007) presents a generally good explanation of the two-period model. Without using calculus, the authors show the socially efficient allocation of the resource. They point out that this allocation results in

MB –MEC growing at the rate of interest. They use the Griffin-Steele diagram to show that the allocation is efficient. They do a great job of emphasizing the role of property rights in the story. If the resource is not privately owned, those with access to it will not take account of the reduction in future scarcity rents that results from using an additional unit of the resource today. When the resource is privately owned, a user today takes that foregone future net benefit into account. They show that the prices implied by the efficient allocation they derived follow the Hotelling rule. They then do a very good job of explaining why the competitive market equilibrium allocation of a non-renewable resource must generate user costs (or scarcity rents) that grow at the rate of interest.

It is, overall, a very good treatment. My concern about Keohane and Olmstead's presentation is how they switch between discussing a single owner's resource and explaining how a competitive market would allocate resources. They start with the case of a single resource owner ("Suppose we own an oil well…" (p. 87)). They seem to assume that this well represents *all* of the resource, because their discussion of the efficient allocation of the resource over time explicitly assumes that selling more today lowers today's price and raises the price in the future. This is confusing, because it may lead the reader to wonder if the owner will take advantage of the market power that the example implies. The early discussion is all in terms of efficiency, raising the question of whether efficiency will conflict with wealth maximization. As the discussion moves to the Hotelling rule, the authors switch from a situation in which "we own an oil well" (for which we are trying to determine the efficient intertemporal allocation—and the amount withdrawn in any period affects the price in that period) to the intertemporal allocation of a non-renewable resource that is owned by many competing entities. In that setting, of course, no resource owner has any impact on the price of the resource in any time period. Having stated, in words, the Hotelling result and having explained why a competitive market will give that result, K&O "test" the theory by plugging in the values they arrived at for the original ("we own an oil well") story and show that the Marginal User Cost does indeed grow at the rate of interest when oil is extracted at the optimal rate over time. But the allocation for the oil well was not arrived at by assuming wealth-maximizing competitive owners. In fact, wealth-maximizing behavior by the owner of the original oil well would give a different (inefficient) time path of extraction. I believe a careful reader of K&O's treatment might come away confused by this story. All confusion would be eliminated if the original story were changed so that it was clear that the problem was to allocate *society's* stock of the resource over time, not an individual owner's stock of the resource. Then, when the Hotelling story is "tested," it would be clear that we are thereby showing the *equivalence* of the efficient allocation and the Hotelling allocation of the nation's resource.

While most of the texts discussed thus fell short of the ideal in some aspect of their presentation of the two-period non-renewable resource story (one didn't explain how a market would be led to the efficient outcome, several relied on calculus, one didn't show that the market outcome was efficient, one seems to suggest that a single resource owner faces a downward-sloping demand curve for its resource and must take into account how its time path of withdrawals would affect prices in different periods), they all got it mostly right. The same cannot be said for the final book considered, by James R. Kahn (2005). Kahn treats this subject in a two-page appendix that employs calculus. The appendix purports to find the dynamically *efficient* allocation of an exhaustible resource (p. 41). However, the mathematical equation Kahn employs to find this allocation is structured to locate the allocation that maximizes the PV of revenue (he assumes a zero MEC) for a *firm* that faces a downward-sloping demand curve in each period. He assumes the demand equation in each period is $p_t = 500 - .5q_t$. The discount rate is 5%. There 100 units of the resource. His equation is

$$PV = (500 - .5q_1)q_1 + [476.2 - .4762(100 - q_1)](100 - q_1)$$

The first term is revenue in period 1 as a function of the amount sold in period 1. The second is the Present Discounted Value of revenue in period 2 as a function of the amount sold in period 2, where the amount sold in period 2 is 100 minus the amount sold in period 1. He maximizes this PV expression by taking the derivative with respect to q_1 and setting it equal to zero. The solution values for q_1 and q_2 are 60.97 and 39.02, respectively, implying prices in the two periods of 469.5 and 480.5, respectively. His arithmetic is correct, but he has solved for the *monopoly* allocation, not the socially efficient allocation. His formulation of the problem takes into account the effect that changing the amount extracted has on the prices in the two periods. This would be the correct way to set up the problem for a wealth-maximizing monopolist, but Kahn claims not to be doing this. The beginning of the appendix explicitly states that he is trying to determine the *efficient* allocation.

Kahn continues his discussion by attempting to illustrate the solution using the Griffin-Steele diagram. As explained above, that diagram (for the zero MEC case that Kahn treats) locates the optimal allocation by finding the intersection of D_1 and the PV of D_2, where the PV of D_2 runs from right to left. This apparatus does indeed illustrate the efficient intertemporal allocation of the resource, but the answer it arrives at differs from

Kahn's answer, since Kahn mis-specifies the problem. Recall that a useful feature of this apparatus is that the intersection represents an allocation where the two quantities add up to the total amount available, and the PV of (P_2 – MEC) = P_1 – MEC. In Kahn's case, since he assumes that MEC = 0, this reduces to PV of $P_2 = P_1$, or $P_2 = (1+r)*P_1 = 1.05P_1$—the Hotelling result. (The quantities would be 73.17 and 26.83, respectively, and the prices would be $463.41 and $486.59, respectively.) But the prices Kahn derived do not meet this condition. P_2 is only 1.023 times as high as P_1. This should not be surprising. A wealth-maximizing monopolist would allocate the resource so that MR_2 – MEC = (1+r)*(MR_1 – MEC). In Kahn's example, $MR_t = 500 – q_t$. Thus, $MR_1 = 439.03$, $MR_2 = 460.98$, and $MR_2 = (1+.05)*MR_1$.

Kahn's presentation of the problem of allocating a non-renewable resource over time falls short in all areas. As noted, Kahn mis-states the problem and derives the wealth-maximizing solution for a monopolist, rather than the socially efficient allocation. Since he does not derive the efficient allocation, he is unable to show that the solution he obtains maximizes the present value of the sum of net benefits over time. He does not introduce the concept of user cost, so he is unable to show the role of user cost in determining the efficient time path of consumption and prices. He does not discuss the competitive equilibrium solution to the problem, let alone show that the competitive equilibrium is efficient. (On p. 42, he refers to "the owners of the coal" but then proceeds through his example to treat the industry as a monopoly or perfect cartel.) He does not mention the Hotelling rule. He uses calculus. While the use of calculus to discuss the problem at hand is not a fatal flaw—several good presentations used it—in Kahn's case, it may have blinded him to his error in setting up the problem. He could have obtained the correct result if he had set up his problem as maximizing the present value of consumer benefits (net of extraction costs). TB_t would equal $500q_t - .25q_t^2$. Had he used this, instead of the expressions for p_t, he would have obtained the standard result. The very fact that the egregious errors in Kahn's treatment (which appears in an appendix) have survived two editions is evidence that the topic is not being covered by most adopters of Kahn's text. This should serve as a warning to authors to put the discussion of this topic in the body of a chapter and to avoid the use of calculus, if possible.

Concluding Comments

Being able to explain the two-period intertemporal allocation problem for the simple case of constant MEC and unchanging demands should be a goal of every student in a course on natural resource economics. While most of the texts reviewed fell short of the ideal presentation in one way or another, all but one gave a satisfactory treatment. That one, the text by James R. Kahn, did not meet any of the goals of a good presentation of this topic. Users of this text should be warned to skip the appendix that discusses this topic and to devise another way to present the economics of a nonrenewable resource.

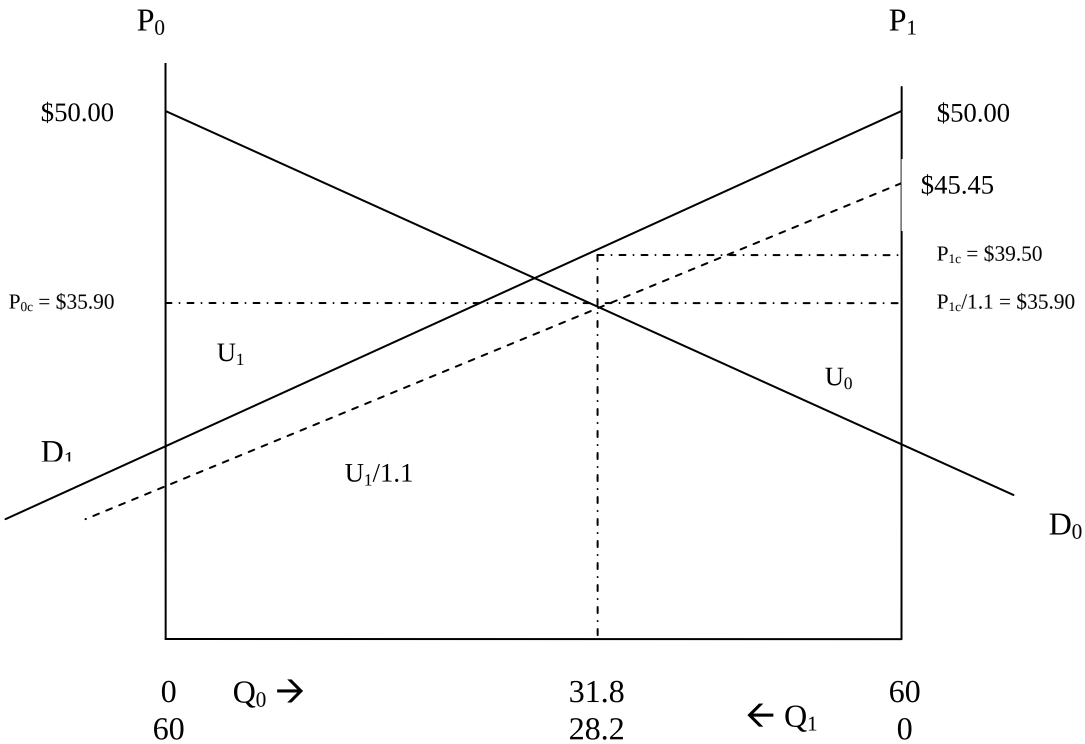

Fig. 1. Solution of the Hotelling price path and production rates (Griffin and Steele, p. 76).

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