LOOK BACK!

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Over the years Word Ways has displayed a varied logological corpus. In this column I revisit forgotten ideas, connect seemingly-disparate concepts, and suggest further investigations.

In the Nov 2007 Word Ways, Jeremiah and Karen Farrell explored the topic of orchard words, groups of heterograms (words with no repeated letters) which can be placed in straight lines on a two-dimensional planar surface. Why orchard words? Each letter can be regarded as a tree planted in an orchard, the objective being to plant n trees in r rows with k trees in each row. Any two rows consist entirely of different trees, or have at most one tree in common planted at their intersection (but it is not required that a tree be planted at the intersection of two rows). What is the maximum value of r for a given k and n? This is discussed in “Tree-Plant Problems” in Martin Gardner’s Time Travel and Other Mathematical Bewilderments (Freeman, 1988).

The linguistic analogue of the horticultural problem requires one to form as many k-letter heterograms as possible out of n different letters assigned to the different trees. It is the purpose of this article to relate this to other kinds of word groups discussed in Word Ways.

The most relevant word group is found in “Dudeney’s Lost Word-Puzzle” in the Aug 1986 issue. Dudeney formed 21 three-letter words out of the 15 letters A through O, with the requirement that any two words could have at most one letter in common. He found ale cab hag fan jam aid oak bed ice foe hen gem kin him jib fig oil con hod jog and mob. If the word jek existed, all possible vowel-consonant pairs would have been represented, so that 21 is the best that one can do. Of course, if “words” like cdj and bjk existed, one could do even better. Dudeney did not impose the geometrical requirement of the orchard problem. It is possible to lay out an orchard of 15 trees with 31 different 3-tree rows, but it is impossible to assign letters so that words can be formed in all cases—nine all-consonant words are needed.

A more restrictive version of Dudeney is achieved by insisting that any two words must have exactly one letter in common. Groups of this nature are discussed in “Word Groups” in the May 1977 issue. These are known as partially-overlapping word groups. Two simple ones are

(6 letters, 4 words) can cot ate one
(7 letters, 7 words) ado ore bar boy yea bed dry

The first of these is equivalent to Farrell’s STREAM example, and the second to his SARDINE one, with the omission of dry.

There are two ways in which Farrell’s examples of orchard words can be generalized. The obvious one is to consider four-letter heterograms; the second is to require that the letters in each row spell out a word directly (not after transposition). What are the largest groups of words for which one can find in these four cases? The following examples almost always use words from the Merriam-Webster Pocket Dictionary; asterisked ones can be found in The Official Scrabble Players Dictionary (Third Edition).
3 letters in order (9 letters, 10 words) Farrell’s MOUSETRAP example with YRP on the top row, EAU on the second, HTN on the third: pry eau* nth yet nay her rat run hap put.

3 letters transposed (11 letters, 16 words) Farrell’s SPECULATION example with YTE on the top row, SNORD across the middle, HAU along the lower edge: yet shy due oat sen dry had sau* nod hoe you ors* are any nth put. This is Problem 213 in Dudeney’s Amusements in Mathematics (Dover, 1958). For 12 letters, 19 words is the theoretical maximum. It appears impossible to find a set of 19 words from the Scrabble dictionary, the problem being that four all-consonant or all-vowel words are needed, but the Scrabble stock is limited to cwm pht nih tsk aye you eau.

This is Problem 213 in Dudeney’s Amusements in Mathematics (Dover, 1958). For 12 letters, 19 words is the theoretical maximum. It appears impossible to find a set of 19 words from the Scrabble dictionary, the problem being that four all-consonant or all-vowel words are needed, but the Scrabble stock is limited to cwm pht nih tsk aye you eau. The 12-letter pattern, apparently discovered by R.H. Macmillan, is found in a 1946 note in the Mathematical Gazette, and is Fig. 137 in the Gardner book.

4 letters in order (11 letters, 6 words) Gardner’s book, Fig. 139 schematically rearranged:

EFT S
efts rocs* ansu dare fond duct
ROCS
ANUS
DD

4 letters transposed (13 letters, 9 words) Gardner’s book, Fig. 143, is diagrammed below. It has not been proved maximal, but no 10-word arrangement has been found. This is Problem 435 in Dudeney’s 536 Puzzles and Curious Problems (Scribner’s, 1967).

U O

B F

N L

A

S V

Y R T E

The words are bony slay trey fuel nave buns fora volt tuba.

Four letters transposed (16 letters, 15 words) This elegant pattern, consisting of a star containing a smaller star containing a pentagon containing a central point, is given in Figure 140 of Gardner’s book; it was originally presented by Dudeney as Problem 21 in The Canterbury Puzzles and Other Curious Problems (Dover, 1958). The points of the outer star consist of the letters JDLNT, the points of the inner star, IUYEO, the points of the pentagon, PCRGS, and the central letter is A. All sixteen words consist of two vowels and two consonants: July jape join deny coda duit* orgy gaun* cure lair tole pyic* soup gies* stay.
It is not likely that sets of four-letter orchard words can be found corresponding to patterns of more than 15 letters, even with the aid of a computer. Gardner exhibits ones containing 19 points in 19 rows (Grinbaum), 20 points in 18 rows (Loyd), and 20 points in 21 rows (Lopow). Unfortunately, one must use several rather rare letters.

The geometric property of orchard words is reminiscent of word squares; each row of an r-by-r square has r intersections with all columns, and no intersections with any other row. For examples, see “Heterogrammic Word Squares” in the Aug 2007 Word Ways. The example above is a 3-by-3 heterogrammic square augmented by the same letter at the end of each row and each column.

INFATUATION

WIN EMMONS
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Infatuation

Coming after several minutes
Of intense concentration, Sid smiled
Long and hard at the ultimate
In beauty staring back at him.
What a perfect thing she was
To contemplate, such a goddess.
Divine? He imagined her to be so.
Did he deserve her? To believe his eyes
Required a leap of faith.
God? Fate? Hope? All of these?
Such a mystery.
God, fate, hope, all of these
Required a leap of faith.
Did he deserve her? To believe his eyes,
Divine he imagined her. To be so!
To contemplate such a goddess!
What a perfect thing she was
In beauty staring back. At him!
Long and hard, at the ultimate
Of intense concentration, Sid smiled,
Coming after several minutes.