DIFFERENCE WORDS

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In his book Beyond Language (Scribner's, 1967), Dmitri Borgmann introduces the concept of difference words. As he points out in Problem 45 (The New Chemistry), the letters of the generating word FORM occupy positions 6, 15, 18 and 13 in the alphabet. The difference between 6 and 15 is 9; between 15 and 18, 3; and between 18 and 13, 5. The letters of the word ICE occupy positions 9, 3 and 5 in the alphabet; therefore, ICE is called the difference word of FORM. For most words, of course, this process leads to a meaningless jumble of letters instead of a difference word.

The above process is unique; that is, each word generates at most a single difference word. However, the inverse process is non-unique; that is, a single difference word can have more than one generating word. To illustrate, consider the generating word CLOT, which has letters occupying positions 3, 12, 15 and 20 in the alphabet. The successive differences of this series are again 9, 3 and 5; in other words, ICE is the difference word for CLOT as well as FORM.

How many different generating words lead to the same difference word? If any possible combination of letters can be counted as a "word", the answer to this question is mathematically derivable. (Non-mathematicians may, if they wish, skip the rest of this paragraph.) Let $x_i$ denote the position of the $i$th letter of the difference word in the alphabet. The number of two-letter combinations leading to the same one-letter difference word is $2(26 - x_1)$, and the number of three-letter combinations leading to the same two-letter difference word is given by the formidable equation

$$2(26 - \min(0, x_1 + x_2)) + 2(26 - x_1 - \max(0, x_2 - x_1))$$

where $\min(x, y)$ denotes the smaller of the two quantities $x$ and $y$, and $\max(x, y)$ denotes the larger of these. To illustrate with a simple example, consider the difference word AX, for which $x_1$ equals 1 and $x_2$ equals 9. Substituting into the above equation, we find that there are $2(26 - 25) + 2(26 - 1 - 23)$, or 6 possible letter combinations. It is not hard to discover that these combinations are AX,
YZB, ZYA, CBZ, ABZ and BAY -- the last combination forming the only generating word. Unfortunately, the formulas corresponding to difference words of three or more letters are quite complex.

A difference word can, of course, be the generating word for a second difference word. For example, the word SET has the difference word NO, and the word NO has in turn the difference word A. No chains longer than this are known; the reader is invited to discover longer chains if he can. Obviously, a generating word of k letters can have a chain at most k links long.

Working in the opposite direction, one can construct difference trees -- first, find all the generating words corresponding to a given difference word; next, regard these generating words as difference words and find all the generating words corresponding to them; and so on. The two largest trees I have been able to cultivate are:

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I
- OX -- PAY
  |   EN -- AFT
  -   OF -- PUG
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 BE
- KIN
  |   BE -- KID -- GRIM
  -   OF -- BIRD
  -   OF -- SLUG
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Does any reader have a greener thumb?

Several logological properties of difference words should be briefly noted. If a generating word is a palindrome, its difference word will also be a palindrome; but the converse statement is not necessarily true. A new generating word - difference word pair can be constructed by reversing the letters in each word. A word with two successive letters the same cannot have a difference word. It is extremely unlikely that a word containing the letter Q can generate a difference word.

I close this article with a list of difference words. To keep this list within reasonable bounds, both the difference word and the generating word were restricted to Webster's Collegiate Dictionary, Seventh Edition.

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a - no, on
I - en, of, ox
ad - hie, him
am - ban, nob
an - tug, vug
as - bat, tub
at - dey, fey
ax - bay
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be - gin, kid, lo - map
kn, sup
em - fan, job
en - aft, pug
he - ald, got, wot
id - rim
if - jag
in - lug
la - con
me - bot
no - set
os - pat
ox - pay
so - tap
to - yet
us - vat
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Thus for word correspondence:

GHANA --> body care
Thus far, I have been unable to find a five-letter difference word corresponding to the above rules; the closest I have come is GHANA - ELDEST, MOSHE - RETAIN and LIKED - FRITOS. Anybody care to try?