THE TWENTY-ONE WORDS

A. ROSS ECKLER
Morristown, New Jersey

Problem: Using any 21 alphabetic letters of your choice, each one exactly five times, construct a group of 21 five-letter words with the properties that (1) any two of the 21 letters being used will be found in exactly one word, and (2) any two of the 21 words will have exactly one letter in common.

This problem, first formulated by Dmitri Borgmann in Problem 122 (Finite Projective Geometries) in Beyond Language (Scribner's, 1967), is undoubtedly one of the most difficult ones in recreational linguistics. Even Dmitri admitted to bafflement: "We have no solution to offer you, good, bad, or indifferent, nor a more detailed statement of conditions applicable to the problem in practice. This is virgin territory, yours for the taking". The purpose of this article is to examine the problem in more detail, presenting two partial solutions and suggesting what is required to obtain a complete solution.

First, let's set the 21-word problem in broader perspective. If one is not constrained to form words out of the letters, but simply assemble them into five-letter groups satisfying the above two conditions, the problem was solved long ago by combinatorial mathematicians:

| a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t |
| a | e | i | m | r | a | k | p | s | l | n | t | h | j | o | q | s | r | t |
| c | f | g | h | q | b | f | j | n | r | e | l | o | s | b | k | m | t | g | p |
| i | j | k | l | q | c | g | k | o | r | c | h | i | n | c | e | j | p | t | c |
| m | n | o | p | q | d | h | l | p | r | d | g | j | m | d | f | i | o | t | d |

Note that each group contains five different letters; thus, one must look for five-letter isograms in solving the problem. This is, in fact, an example of a balanced incomplete block design, which are much used by statisticians in designing experiments. Similar balanced incomplete block designs can be constructed using 13 different letters arranged in groups of four, and 7 different letters arranged in groups of three. The linguistic analogues of the latter two designs were first examined by Ronald C. Read in "Soup, Fish and Finite Geometries" in the February 1963 issue of Recreational Mathematics Magazine. The 7-word problem is trivial; a typical solution is DRY ORE BOY ADO BAR AYE BED. The 13-word problem is considerably more difficult; the first complete solution was given in Beyond Language, and a solution using only words from Webster's Second and Third was reported in the May 1972 Word Ways: AITU HIES HURL MOUE MYTH OILY OPAH PELT PRIM PUYS SLAM SORT YEAR. A solution confined to Webster's Collegiate Dictionary is very likely impossible.
Why is the 21-word problem so difficult? To begin with, one must select all but five letters of the alphabet, which means that (unlike the 13-word and 7-word problems) moderately rare letters must be used. It seems reasonable to exclude J, Q, Z and X from the solution without further thought, as five-letter words with these letters are decidedly rare. The choice of the fifth letter to eliminate is a little less obvious, but V is probably the best choice because five-letter words using V typically employ more vowels than five-letter words using K, B, F or W (these letters readily forming such bigrams as fr, fl, br, bl, wh, wr, ck, lk, nk, rk, sk).

The difficulty of the 21-word problem is increased by the restrictions that must be placed on the distribution of vowels among the words. If AEIOUY are taken as vowels, there are 30 vowels to be distributed among the 21 words, an average of 1.4 vowels per word. However, if one examines the combinatorial letter-patterns presented above, it quickly becomes clear that the vowels cannot be allocated evenly. There are, in fact, six possibilities, given in the table below.

<table>
<thead>
<tr>
<th>Number of Words Having</th>
<th>Vowels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>QUARTS</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

How is this table read? Consider, for example, the vowel allocation given on the first line. If the vowels are identified with the letters Q, U, A, R, T and S in the design on the preceding page, 15 of the 21 letter-groups have one vowel, 5 have two vowels and 1 has five vowels.

The first two vowel allocations are the ones of greatest interest to the logologist. To solve the 21-word problem, one must locate either a five-vowel word or two no-vowel words with only one letter in common, a seemingly impossible task. As far as I know, there are no all-vowel five-letter isograms in Webster's Second or Third, and only one all-consonant isogram, CRWTH, in these two dictionaries. A list of approximately 57,000 five-letter words, compiled many years ago by National Puzzlers' League members to aid in the construction of forms (word squares, diamonds, etc.) and taken from two dozen turn-of-the-century dictionaries and gazetteers, revealed one all-vowel isogram, OYAUE, and one more all-consonant isogram, TWNGS (which unfortunately has two letters in common with CRWTH). The use of OYAUE presents the solver with the rather difficult problem of finding four words, three with Y as the only vowel and one with the pair of vowels Y and I, which exactly use up the consonants. It seems more desirable to locate a five-vowel word that permutes the letters AEIOU. The March 1971 issue of The Enigma mentioned a passage from Dante's The Convivio, IV, 6, which discusses The Word: "a verb, dropped very much out of use in Latin, which signifies 'binding words together' to wit aueio." Coming
from a parent language of English, a Latin word is acceptable until something better is found. As for all-consonant words, Darryl Francis pointed out the existence of the hamlet of BWLCH in the Times Atlas of the World (located about 10 miles northwest of Abergavenny in Wales) as well as the word TRWMP, a 14th-century variant of trump in the Oxford English Dictionary, fulfilling the requirement of two no-vowel words with only one letter in common.

I first examined the possibility of obtaining a solution based on AUEIO. Each of the other twenty words needed in the solution consists of four letters taken from the set BCDFGHKLMNPRSTWY, plus one letter taken from the set AEIOU. There are a total of 9050 ways in which this can be accomplished. In order to determine quickly which of these five-letter groups corresponded to words, I sorted out all five-letter isograms from Webster's Second (conveniently given in Lee Keith's 1952 booklet) and from the previously-mentioned formist list into a table containing these 9050 categories; a total of 3426, or about 38 per cent, were actually occupied by words. Using this table, I then determined which letter-pairs were represented by the smallest number of different words. FB and BP were the rarest pairs, with about 20 words apiece; other unusual letter-pairs included FP, FM, WU, GK, FG, CG, BW and FW.

Since every letter-pair must appear in a word in the solution, I adopted the strategy of first forming words containing the unusual letter-pairs, leaving the commoner ones until later. To get off to a fast start, I checked the list for words containing two unusual letter-pairs (pflug, flamb, funbl, wurmb, wurbs, kampf, flegm, flump, frump, greck). I soon zeroed in on KAMPF as the most desirable starting word; it combined successfully with about 125 different combinations of FB and BP words (kampf-blifs-phoby, kampf-forby-blupt, kampf-befts-proby, etc.). I followed up all these leads, adding words from the lists of other unusual letter-pairs until I could proceed no further. Finally, I took the longest of these sequences (typically, those with seven or eight words) and attempted to add as many common-letter-pair words as possible. When I had exhausted the words in the 9050-category sort, I looked for various arrangements of the missing letter-groups in the Oxford English Dictionary, telephone directories and the like.

The work briefly described in the preceding two paragraphs took a number of evenings and weekends of work spread out over several months. At the end of that time, the best list I had devised contained 16 of the 21 words:

AUEIO binding words together (from Dante's The Convivio)
BAWND swollen (Halliwell's Provincial Dictionary, 1881)
BERGK German linguist and translator (Phillips' Index of Biographical Reference, 1871, 1882, 1893)
BLIFS believes (Webster's First Edition)
CLARY an aromatic herb and ornamental (Webster's Third Edition)
DOCKS
FRONT
FUDGY awkward (Webster's Second Edition)

At this juncture, BWLCH and TRWMP, almost immediately both words with no vowels, were constructed as words with no vowels, and W, G, W, and C, common with these, formed a group of words: gaund-gowks, and the like.

All this infuriated me, and I continued to work on various hours, upon checking the lists of words, I discovered

BWLC
COATI
FORBY
GAWFS
HOMES
KAUR
KLEFT
KNOWD
MADLY
NYGHT
PBRAN
PYCKS
PLOUG
SNIRL
STUBD
TRWMP

YOWIE.
At this juncture, Darryl Francis came up with the pair of words BWLCH and TRWMP and urged me to try for a solution based on them. Almost immediately I found that I was very constrained in adding words to these. To begin with, I needed a word with four vowels and the letter W, but could only locate OIEWA, YEOWA, YUWIE and IOWA Y in the formist list. Given the four-vowel word, I then found it necessary to construct a word out of one of the unused vowels, the letter W, and three of the consonants FDKNGS, and another word out of the other unused vowel, the letter W, and the remaining three consonants. This proved to be possible only with YUWIE, leading to five allowable pairs of words: gawfs-knowd, kwang-dowfs, gownd-wakfs, dwang-fowks, and fawnd-gowks. Continuing on, I next sought to form two words out of eight different consonants and two Ys, these words having one letter in common with each of the five words already selected for the sequence.

All this cerebration was necessary before I could begin to fit the nine remaining one-vowel words into the solution. After additional tedious hours, I finally came up with a 17-word solution. Unfortunately, upon checking the words from the formist list with Palmer Peterson, I discovered that YUWIE was nonexistent, apparently being a misspelling of YOWIE. The resultant list is given below:

```
BWLCH a village in Wales (Times Atlas of the World)
COATI a tropical American mammal (Webster's Third Edition)
FORBY besides, except (Webster's Third Edition)
GAWFS cheap red apples (Webster's First Edition)
HOMES KHAUR var. of gur (Webster's Second Edition)
KLEFT var. of kleft (Oxford English Dictionary)
KNOWD the gray gurnard (Webster's First Edition)
MADLY NYGHT var. of night (Oxford English Dictionary)
PEBAN South American linguistic family (Webster's Second Edition)
PYCKS var. of pitch (Oxford English Dictionary)
PLOUG Danish poet (Phillips' Index of Biographical Reference, 1871, 1882, 1893)
SNIRL twist, snarl (Webster's Third Edition)
STUBD reformed spelling of stubbed (Webster's Second Edition)
TRWMP var. of trump (Oxford English Dictionary)
```

bcmtu, cefhw, cglnp, delpt, dhmm
Can the 21-word problem be solved? Probably not, if one is confined to Webster's Second and the formist list. There exist at least three sources of words, however, that have not been systematically mined for five-letter isograms: (1) the Oxford English Dictionary, (2) the Official Standard Names Gazetteers (issued for various countries by the U.S. Board on Geographical Names), and (3) surnames in United States telephone directories. The latter source may be especially rich; there are approximately 150,000 surnames in Social Security files spelled with five letters.

Even if lists of five-letter isograms were compiled from these sources, difficulties remain. It is impossible to check more than an infinitesimal fraction of the likely sequences of words, no matter how clever one is in eliminating unprofitable lines of inquiry. The task is ideally suited for a digital computer which can search through alternatives in a small fraction of the time required by humans. If it were programmed to follow up promising leads and abandon unfruitful ones early, I believe that it might be able to find a solution to the 21-word problem.

AN ALMANAC OF WORDS AT PLAY

This is the title of a major new book on wordplay, written by Willard Espy and published by Clarkson N. Potter (distributed by Crown Publishers). Available in two forms, a hardcover edition for $12.95 and a paperback for $6.95, this book is arranged in the form of an almanac, with one entry for each day of the year. It is a galimaufry of every conceivable type of linguistic recreation, past and present: odd words, light verse, parodies, anagrams, palindromes, fractured English, graffiti, typos, coined words, epitaphs, Black English, Pidgin English, rhopalic verse, lipograms, malapropisms, rebuses, spoonerisms, macaronics, oxymorons, acrostics, double-dactyls and much, much more. Some of the best pieces in the book are Espy's own light verses, a number of which have been featured in Word Ways during the past couple of years (for the latest, see 'The Poet's Corner' in this issue). Run, do not walk, to your nearest bookstore for a copy!