

CYCLE STRUCTURE OF WORDS

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The letters of a word can be numbered alphabetically, i. e., the letter of the word occurring earliest in the alphabet can be assigned the number 1, the next letter 2, etc. For example, using this scheme, the word PROBLEM would be converted into 6 7 5 1 3 2 4 since the B is the earliest letter in the alphabet occurring in the word, E is the second, etc. If a word contains repeated letters these may be numbered in increasing order from left to right; thus, the word DESCEND would be numbered 2 4 7 1 5 6 3, the two D's receiving numbers 2 and 3, and the two E's the numbers 4 and 5. However, in what follows, only words with no repeated letters will be considered.

Once a word has been converted into a set of numbers, it can be the subject of many mathematical operations. We propose to consider the cycle structure, that is, the way the numbers chain together by position. The numbers 6 7 5 1 3 2 4 corresponding to the word PROBLEM represent a pair of cycles, one of length two (3 5) and the other of length five (6 1 4 7 2). This is derived by noting that the numbers 5 and 3 are in third and fifth positions, interchanging with each other, and the remaining numbers form a chain in the following manner: the number 6 is in the first position, the number 1 is in the fourth position, the number 4 is in the seventh position, the number 7 is in the second position, and the number 2 is in the sixth position, completing the cycle.

A number of nine-letter words containing no repeated letters were studied from the point of view of their cycle structure. There is nothing special about nine-letter words; words of any length can be studied from a cycle structure standpoint. However, this length was selected because it was the longest which did not require two-digit numbers.

The first group is composed of one-cycle words; the next groups exhibit examples of the four different ways in which nine-letter words can form exactly two cycles: cycles of lengths 1 and 8, of lengths 2 and 7, of lengths 3 and 6, and of lengths 4 and 5. The extension of the table to words containing three or more cycles is left as an exercise for the reader. Note that the word containing nine cycles, all of length 1, is a word in which the letters are in alphabetical order; no such nine-letter word is known.

As it is immaterial where a cycle starts, all cycles given below arbitrarily begin with their lowest numbers.

One Cycle Words

discharge (1 6 5 8 7 2 4 9 3)
 education (1 5 7 8 6 9 3 2 4)
 excursion (1 3 7 6 5 8 4 9 2)
 logarithm (1 4 6 9 7 2 3 8 5)
 masculine (1 2 4 7 8 3 9 5 6)

ownership (1 4 3 8 6 9 2 7 5)
 something (1 4 7 2 9 5 3 6 8)
 sunflower (1 8 2 4 3 5 6 9 7)
 yardstick (1 2 8 6 3 4 7 5 9)

Two Cycle Words

certainly (9)(1 5 8 4 6 7 3 2)
 complaint (9)(1 6 8 4 5 3 7 2)
 grandiose (7)(1 3 9 8 2 5 6 4)
 greyhound (6)(1 9 4 5 8 7 2 3)
 hailstone (7)(1 2 9 6 8 5 4 3)
 housemaid (6)(1 7 2 9 3 5 8 4)
 inoculate (8)(1 7 3 9 5 6 2 4)
 labyrinth (8)(1 2 3 9 4 6 7 5)
 longevity (9)(1 5 3 7 8 6 2 4)

boulevard (4 5)(1 7 8 3 9 6 2)
 hyperbola (5 8)(1 9 2 6 7 3 4)

horseback (2 6 9)(1 7 3 8 4 5)
 nostalgic (1 5 6)(2 9 4 8 3 7)
 pseudonym (1 5 6)(2 3 9 8 4 7)
 submarine (2 3 9)(1 5 4 7 6 8)

facetious (1 2 3 4)(5 6 7 9 8)
 livestock (2 4 9 3)(1 8 6 7 5)
 porcelain (3 5 6 9)(1 7 2 4 8)

observant (2)(1 7 3 4 8 9 6 5)
 oligarchy (9)(1 5 3 4 8 6 2 7)
 overnight (5)(1 3 8 9 2 7 4 6)
 pachyderm (8)(1 2 3 6 9 5 4 7)
 parchment (9)(1 2 4 5 6 8 3 7)
 patronize (5)(1 2 9 8 3 7 4 6)
 rhapsodic (6)(1 3 7 4 2 9 5 8)
 spearmint (9)(1 4 6 2 3 7 5 8)
 sprightly (9)(1 5 2 6 3 4 8 7)

inspector (7 9)(1 6 4 2 5 8 3)
 stockyard (1 7)(2 4 5 3 9 6 8)

slaughter (4 6 9)(1 3 5 2 8 7)
 symbolize (2 9 8)(1 4 6 5 3 7)
 tenacious (2 5 3)(1 4 6 7 9 8)

vehicular (2 5 4 3)(1 8 6 7 9)
 workbench (1 5 4 9)(2 8 3 6 7)