WORD GROUPS

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Some words, like DEIFIED, have patterns that immediately attract the eye; others, like SCINTILLESCENT, possess more subtle charms (each letter appears exactly twice). In recent years, logologists have discovered a number of remarkable word groups; even though the individual words are quite ordinary in appearance, taken together they exhibit unsuspected symmetries of various types. To show what is possible, the letters of seven three-letter words in the column at the left have been rearranged in a square array:

ADO	A		D		O		
ORE				E	O	R	
BAR	A	B				R	
BOY		B			O		Y
YEA	A			E			Y
BED		B	D	E			
DRY			D			R	Y

Each word is an isogram; that is, it contains no repeated letters. Collectively, the seven words consist of a total of seven letters, each used three times. Further perusal of the array reveals that any word has exactly one letter in common with any other word -- for example, ADO shares an A with BAR and YEA, a D with BED and DRY, and an O with ORE and BOY. Another property of the array is a bit more subtle. There are a total of 21 different ways one can pick two letters out of the set ABDEORY: ab, ad, ae, ao, ar, ay, bd, be, bo, br, by, de, do, dr, dy, eo, er, ey, or, oy, ry. Each of these pairs occurs in exactly one word -- ab in BAR, ad in ADO, ae in YEA, and so on to ry in DRY.

This pattern was first exhibited by Ronald C. Read in "Soup, Fish and Finite Geometries" in the February 1963 issue of Recreational Mathematics Magazine, and later appeared in Dmitri Borgmann's Beyond Language (Scribner's, 1967) as problem 122.

Balanced Word Groups

Word groups with these properties have been referred to as finite projective geometries, a term which reflects the underlying mathematics but says nothing about the word relationships. Because it is difficult to discover a short phrase specifically describing all the properties exhibited above, the term heading this section has been adopted. These designs are balanced (unlike others to be introduced presently) because the number of different letters used equals the number of words in the group (equivalently, the number of repetitions of each letter in the group is equal to the word length).

Borgmann's book pointed out that only four different patterns of this type were possible: 3 words of two letters, 7 words of three letters, 13 words of four letters, and 21 words of five letters. Since the number of words is equal to the number of different letters that must be used, it is clear that the longer word groups are much harder to find than the shorter ones. A three-word list is given by BE BY YE. The following thirteen-word group, containing words all found in Webster's Third Unabridged and above the line in Webster's Second, was constructed by Mary J. Hazard of Rochester, New York and published in the August 1972 Word Ways:

CITY		C			I						T		Y
CLAD	A	C	D			L							
CONE		C		E				N	O			W	
CWMS		C					M			S			
DIME			D	E	I		M						
DOTS			D						O	S	T		
IOWA	A				I				O			W	
MANT	A						M	N			T		
MOLY						L	M		O				Y
NILS					I	L		N		S			
WELT				E		L					T	W	
WYND			D					N				W	Y
YEAS	A			E						S			Y

The 21-word group appears to be impossible to form without a huge word-list of five-letter isograms and a high-speed digital computer to search possibilities rapidly. It can be shown that such a word group must have either one all-vowel word (AEIOUY counted as vowels) or two all-consonant words with only one letter in common. CRWTH is apparently the only all-consonant five-letter isogram in Webster's Second or Third, but a better starting-point is BWLCH (a Welsh village in the Times Index-Gazetteer) and TRWMP (an obsolete variant of <u>trump</u> in the Oxford English Dictionary). By drawing on a wide variety of word sources, including geographical names and surnames, solutions with 16 of the 21 words have been located; the reader is referred to the November 1975 Word Ways for details.

Can other balanced word groups be found? Yes, if one generalizes the conditions a bit to allow (1) words to have m letters in common, not just one, and (2) pairs of letters to appear together in n words, not just one. Consider, for example, the word group:

NEAT	A	E	N		T
SANE	A	E	N	S	
NEST		E	N	S	T
TANS	A		N	S	T
SEAT	A	E		S	T

Here, each word has three letters in common with each other word, and each pair of letters appears together in exactly three words. This is known as a Baltimore transdeletion, a term coined by L. M. N. Terry in 1904, according to A Key to Puzzledom (1906), an early publication of the Eastern Puzzlers' League. It is easy to see that analogous word groups can be constructed for words of any length; the number of words is always one more than the number of letters in each word:

5-letter words: rates, caste, crest, carts, cares, crate
6-letter words: splint, plants, pliant, paints, plains, instal, plaits
7-letter words: stinger, gaiters, retains, seating, strange, ratings, granite, erasing
8-letter words: trinodes, notaries, intrados, tornades, asteroid, strained, sedation, rationed, donaries
9-letter words: stercolin, relations, contrails, consertal, creations, sectorial, larcenist, sectional, crotaline, censorial

Yet other balanced word groups can be constructed. The simplest one that is neither a finite projective geometry nor a Baltimore transdeletion is given below:

SNARL	A					L	N		R	S	
NORTH				H			N	O	R		T
CLINT		C			I	L	N				T
LATHE	A		E	H		L					T
LOCHS		C		H		L		O		S	
CHAIR	A	C		H	I				R		
CANOE	A	C	E				N	O			
OSTIA	A				I			O		S	T
OILER			E		I	L		O	R		
SHINE			E	H	I		N			S	
CREST		C	E						R	S	T

Each word has two letters in common with each other word, and each pair of letters appears in two different words. The complementary word group, one which uses the same group size but a different word length (the two different word lengths always sum to the group size), is given below:

PONDER		D	E		N	O	P	R			
AUDION	A	D		I	N	O					U
URSINE			E	I	N			R	S		U
OUSTED		D	E			O			S	T	U
PISTON				I	N	O	P		S	T	
PEANUT	A		E		N		P			T	U
PAROUS	A					O	P	R	S		U
STRAND	A	D			N			R	S	T	
PUTRID		D		I			P	R		T	U
ASPIDE	A	D	E	I			P		S		
AREITO	A		E	I		O		R		T	

A corresponding pair of complementary word groups consisting of 15 eight-letter words and 15 seven-letter words has not yet been constructed; another complementary pair to try is 16 ten-letter words and 16 six-letter words.

The word group NAIL, SALE, SINE, LEND, IDEA, SAND, SLID is complementary to the group of seven three-letter words introduced at the beginning. However, a word group consisting of 13 nine-letter words, complementary to the word group constructed by Mary Hazard, has not been found.

Word Groups Containing All Pairs of Letters

To find additional word groups, one must eliminate one of the special properties described above. If one waives the requirement that each word should have exactly m letters in common with each other word, but retains the requirement that each possible pair of letters appears in exactly n words, one is led to a large number of interesting word groups.

In fact, one can construct complete word groups for any choice of word length and number of alphabetic letters by the simple expedient of forming i-letter words out of all possible subsets of j letters. For example, there are ten ways one can select three letters out of a stockpile of five different letters:

YEA	A	E			Y
YES		E	S		Y
YET		E		T	Y
SAY	A		S		Y
TAY	A			T	Y
STY			S	T	Y
SEA	A	E	S		
TEA	A	E		T	
SET		E	S	T	
SAT	A		S	T	

Each pair of letters appears in exactly three words (for example, ae in YEA, SEA and TEA). Note that the overlap between different words is irregular; YEA and YES have two letters in common, but TEA and STY only one.

Complete word groups which are not Baltimore transdeletions are difficult to construct in general because there are too many different alternatives -- in general, there are $j!/i!(j-i)!$ different ways of selecting subsets of i letters out of a stockpile of j letters to form words. The following word groups have been found:

4 letters out of six: peai, aine, itea, atip, anti, pain, pate, pale, pane, pite, pile, pine, pant, pent, pint
5 letters out of seven: reina, irate, anise, tinea, taise, raise, tarns, stern, trins, astir, aster, rites, stein, stain, antes, rinse, saner, rains, niter, antre, train
6 letters out of eight: string, streng, grants, reigns, tigers, ingest,

tinger, insert, astern, argent, grates, agents, angers, strain, gratis, giants, grains, rating, satire, tisane, staige, arisen, agrise, easing, retain, triage, eating, regain

The last word group is slightly defective; one word, STAIGE, was taken from Webster's First Unabridged.

Fortunately, word groups exist that contain all letter-pairs the same number of times but do not use all possible subsets of letters. The simplest example of such a word group is illustrated below:

	A	E	P	R	T	Y
YEA	A	E				Y
PER		E	P	R		
YET		E			T	Y
PAY	A		P			Y
PRY			P	R		Y
TAP	A		P		T	
ARE	A	E		R		
RAT	A			R	T	
TRY				R	T	Y
PET		E	P		T	

Each pair of letters appears in exactly two different words; only half of the 20 possible three-letter subsets of AEPRTY have been used in the word group. These word groups are closely related to balanced incomplete block designs, geometrical patterns used by statisticians to lay out experiments. These designs were identified by mathematicians many years ago, making it easy to select word groups to construct. There are a considerable number of these designs for which word groups can be constructed. The number in parentheses following each word group indicates the fraction of a complete word group that is utilized in the design:

3-letter words: emu, thy, ago, tau, gym, hoe, you, ham, get, hug, tom, yea (1/7)

4-letter words: idea, iota, dote, nolt, lend, nail, aloe, tald, tile, lido, dint, Ione, dona, neat (1/5)
torn, ions, sent, pose, neat, spar, aire, Ateo, porn, anis, rest, opie, pant, soar, rein, spit, trip, iota (1/7)
rent, dots, darn, duns, stir, aitu, Aino, dieu, Osea, roue, date, sine, roid, sura, unto (1/14)

5-letter words: stond, sotie, intro, stair, radio, trade, Diane, rends, aeons, arose, Donat, tides, irone, tarns, adios, doter, tinea, rinds (1/7)
irate, dints, snort, danli, tonal, stead, anode, trild, rains, idose, aliso, lords, toile, slent, irone, laser, Troad, lernd (1/14)

6-letter words: ostein, adorns, tirade, storid, astern, Oneida, rinsed, ration, estado, rodent, ariose, dinast (1/7)
aspine, trepid, strand, sprint, teopan, Portia, ordain, ditone, adopts, Sadite, ariose, spored, tenors, pander, poinds (1/14)

For words of seven letters or more, suitable balanced incomplete block designs are rare, and no word groups based on them have been found.

Partially Overlapping Word Groups

Yet more word groups can be constructed if one waives the requirement that each possible pair of letters appears in exactly n words, but retains the requirement that each word should have exactly m letters in common with each other word. A knowledge of balanced incomplete block designs facilitates the search for these word groups as well.

One sequence of one-letter overlapping word groups, closely related to some discussed in the next section, has the additional property that each letter appears in exactly two words:

CAN	A	C		N		
COT		C			O	T
ATE	A		E			T
ONE			E	N	O	

SCAN	A	C					N			S
SORE			E					O	R	S
COIL		C		I	L			O		
MARL	A				L	M			R	
MINE			E	I		M	N			

WRIST					I						R	S	T		W
WHOLE			E	H		L			O						W
CHARM	A	C		H			M				R				
COUNT		C						N	O				T	U	
PLAIN	A				I	L		N		P					
SPUME			E				M			P		S		U	

WHUMPS								H				M			P		S		U	W	
WICKED			C	D	E				I	K										W	
BLIGHT		B					G	H	I		L							T			
FLUNKY						F				K	L		N						U		Y
DOGNAP	A			D			G						N	O	P						
EMBRYO		B			E							M		O		R					Y
CRAFTS	A		C			F										R	S	T			

The final word group, by far the most difficult, was constructed by Mary Hazard and published in the May 1972 Word Ways. Note that each letter is found in a different pair of words -- in fact, all possible word pairs are uniquely characterized by letters.

Many other partially overlapping word groups are possible, but these all have three or more repetitions of letters in different words, or a higher degree of overlap:

4-letter words: slam, more, mind, neat, told, stir, bard, snob, bile (one-letter overlap)

5-letter words: ogled, grape, poise, grids, plaid, solar
(two-letter overlap)

6-letter words: litany, curate, clypes, adopts, coined, purins, dourly (two-letter overlap)
ignore, phrase, plight, talons (two-letter overlap)
rating, dental, glider (three-letter overlap)
grinds, mating, grates, remand, misted (three-letter overlap)
atones, ratios, retina, senior (four-letter overlap)

8-letter words: sterling, oriental, tangelos, seraglio, organist (six-letter overlap)
ensiform, platform, panelist (four-letter overlap)

9-letter words: goldcrest, nostalgic, declaring, ordinates (six-letter overlap)

10-letter words: canephorus, cingulated, droplights (five-letter overlap)

When the overlap is one less than the number of letters in the word, a Baltimore transdeletion (discussed earlier) occurs. Certain combinations of word-length and overlap are mathematically impossible; for example, one cannot find a group of five-letter words which all have three-letter overlaps.

Possible combinations for which no word groups have yet been found include a group of 15 six-letter words with two-letter overlap, a group of 14 seven-letter words with three-letter overlap, a group of 14 eight-letter words with four-letter overlap, and a group of 12 eight-letter words with five-letter overlap.

A Card Trick Mnemonic

In a well-known card trick, the mathemagician deals ten pairs of playing cards face down and invites the subject to look at the face values of one pair while the mathemagician's back is turned. The mathemagician then deals the cards out in a four-by-five array with card faces upwards. When the subject is asked to say in which of the four rows his two cards appear, the mathemagician immediately identifies the two cards.

The successful execution of this trick depends upon the fact that there are exactly ten different ways in which a pair of cards can be distributed among four rows: both cards in row 1, cards in rows 1 and 2, cards in rows 1 and 3, . . . , both cards in row 4. The mathemagician uses the mnemonic BIBLE, ATLAS, GOOSE and THIGH to aid in placing the cards in the four rows: the first pair of cards occupy the positions of the B's in the first word, the second pair of cards occupy the position of the I's in the first and fourth words, and so on.

The mathemagician's mnemonic is closely related to the first word group at the start of the preceding section, as the following diagram illustrates:

BIBLE	E		I	L			BB
ATLAS				L	S	T	AA
GOOSE	E	G			S		OO
THIGH		G	I			T	HH

The three-letter words have been enlarged to five-letter words by a pair of repeated letters, different for each word.

This mathemagical trick can be readily expanded to 15 or 21 pairs of cards by using other mnemonics: LIVELY, RHYTHM, MUFFIN, SUPPER, SAVANT and MEACOCK, RODDING, GUFFAWS, TWIZZLE, RHYTHMS, KNUBBLY. Too bad the alphabet doesn't have 28 letters, so that logologists could search for the next possible word group!

A NEW JOURNAL OF CRYPTOLOGY

Cryptologica, a quarterly journal dealing with cryptology in all its forms -- mathematical, computational, literary, historical, political, military, mechanical, archeological -- began publication with a 100-page issue in January 1977. In this issue, editor David Kahn (of The Codebreakers) gives a fascinating analysis of the idiosyncrasies of David Shulman's annotated bibliography of cryptology, and editor Brian Winkel describes how a student in his cryptology class cracked a cipher that Poe, back in 1841, had declared to be a "jargon of random characters, having no meaning whatsoever" -- a faulty judgment unchallenged by such expert cryptographers as William Friedman of Japanese purple code fame! (The cipher turned out to be a complaint by G. W. Kulp about the delay in receiving his copy of Alexander's Weekly Messenger.) Articles range from research papers to surveys, personal accounts, book reviews, educational notes and problems; subscription is $5 for the first issue (Aegean Park Press, PO Box 2837, Laguna Hills, California 92653) or $16 per year (Cryptologia, Albion College, Albion, Michigan 49224). To delay would be folly.