TOWARD MORE EFFICIENT NUMBER MNEMONICS

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As pointed out by Ross Eckler in the Nov 2008 Word Ways ("Mnemonics for Number Sequences", p. 297), the well-known type of mnemonic which uses the length of successive words to represent a sequence of decimal digits (with a ten-letter word for each occurrence of zero) is not particularly efficient. If the sequence of digits being represented has a uniform probability distribution (as is the case for the digits of \( \pi \) and \( e \), for example) then in the long run this scheme will have an "inefficiency ratio", defined as \((\text{total number of letters used})/(\text{number of digits represented})\), of 5.5. This is pretty far away from 1, which could be considered ideal in some sense.

Here is a poem that captures the first 134 digits of \( \pi \) with somewhat better efficiency.

Darkness: heavy, dull, silky but somehow grotesque,
Interred within a frozen cell
Lined in macabre skin-framed pallets.

He wipes blood, old as an ache,
Pokes at a fly in disgust.
Chimes urge religious locals to vows of love,
Knees showing a form of loyalty he privately lacks.

Crammed, heavily bound, he heaves
A calm death-mocking word,
Pushing closer to tears.
Quiet scrapes over roads by shrill axles die,
Soundless as a scarecrow.

Voices of bygone folks flow swiftly
Over grave, granite, or greenwood,
Dying in a December sky.

To extract the digits of \( \pi \) from this text, follow these simple rules:

(1) Take each word of three or more letters from the text, in order.
(2) Extract two digits from each word ("first digit" and "second digit") like this:
   - Calculate the word’s score in the game of Scrabble by adding up the score of the individual letters. The right-most digit of the word score is the first digit.
   - Add up the numerical values of the letters (using A=1, B=2, C=3, etc.). The right-most digit of this sum is the second digit.

Here is how this works out for the first few words of the poem:
<table>
<thead>
<tr>
<th>Word</th>
<th>Scrabble score (= first digit)</th>
<th>Letter sum (= second digit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DARKNESS</td>
<td>$2+1+1+5+1+1+1+1 = 13$</td>
<td>$4+1+1+8+11+14+5+19+19 = 91$</td>
</tr>
<tr>
<td>HEAVY</td>
<td>$4+1+1+4+4 = 14$</td>
<td>$8+5+1+22+25 = 61$</td>
</tr>
<tr>
<td>DULL</td>
<td>$2+1+1+1 = 5$</td>
<td>$4+21+12+12 = 49$</td>
</tr>
<tr>
<td>SILKY</td>
<td>$1+1+1+5+4 = 12$</td>
<td>$19+9+12+11+25 = 76$</td>
</tr>
</tbody>
</table>

with the underlined right-most digits making 3,1,4,1,5,9,2,6, the first eight digits of $\pi$. And how efficient is this poem? With 421 letters and 134 digits, its inefficiency ratio is 421/134, which equals...(wait for it)...3.141, or approximately $\pi$. Needless to say, this value was achieved deliberately.

Admittedly, this approach strains the definition of “mnemonic” a bit, in that it is not possible to rattle off the digits of $\pi$ particularly quickly with a mnemonic of this type. However, the rule for extracting the digits is simple enough that one can do so easily with the aid of pencil and paper (or chalk and wall of prison cell, depending on the circumstances).

Having achieved an inefficiency of around 3, I decided to see how close I could approach 2 while still remaining fairly comprehensible. The eight lines below contain 162 letters and encode the first 81 digits of $\pi$, for an inefficiency ratio of exactly 2.0.

Frigid bit of new stone
poised on cystic skin;
Dead stares of ill wisdom,
a sickly lark near future altars,
patrons left madly unkempt.
Six windows aid to deify
pupils' views of iron,
yet gold hinges weep.

The rules used here are an extension of those from the previous poem. Specifically:

- As before, one- and two-letter words are ignored.
- If a word has four or five letters, then the same procedure as before is followed, except that the extracted digits are reversed (first digit = letter score, second digit = Scrabble score).
- If a word has three letters or more than five, then two digits are extracted as described in the previous rule, plus a third digit is produced by taking the right-most digit of the sum of the binary Morse Code values of the letters. The “binary Morse Code value” for a letter is obtained by taking the Morse Code symbol for the letter, replacing each dot with 0 and each dash with 1, and interpreting the result as a binary number. So, for example, B = - - - - which becomes 1000 in binary, or the value 8.

For instance, the first word, FRIGID, falls under the third subrule above, so it produces three digits: from the letter sum $6+1+8+9+7+9+4 = 53$ (=3, the right-most digit), from the Scrabble score $4+1+1+2+1+2 = 11$ (=1), and from Morse Code $2+2+0+6+0+4 = 14$ (=4), giving 314.

How much lower can you go? Readers are invited to take up this challenge.