## THE 4-SET PROBLEM

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23751 different sets of four letters can be formed from the 26 letters of the alphabet, if order is ignored and repetition allowed. (This is the same as the number of ways of selecting four different objects from a group of 26+4-1.) Is it possible, for each set, to find a word containing its letters?

A few special cases have been investigated before. Dmitri. Borgmann in the February 1969 Word Ways considered the sets with all letters the same, from AAAA anabata to ZZZZ pizzazz. Subsequent research, up to February 1983, has left no holes but XXXX, for which we have nothing but complicated, un-English-looking chemical terms, 'xoxxoxo' from Finnegans Wake, and coinages like the adjective 'sixty-six-sixty-sixths'. In the February 1971 Word Ways Murray Pearce looked at the sets of four consecutive letters, from ABCD cabda to ZABC zebraic. He found dictionary examples for all but three, which we here fill in from other sources: UVWX Vireux-Wallerand (populated place, OSFR), VWXY Xsmwdribvnwlxy (play title, NUCL), and WXYZ Xiawaziyu (town, NIA). (Abbreviations references are explained in the bibliography.) In the February 1972 Word Ways Ralph Beaman gave WXYZ waxy maize, from NI, but we restrict ourselves in this project to terms without internal spaces. Dmitri Borgmann asked in Beyond Language (Scribner's, 1965) and later in Word Ways about JQXZ, the set containing the four rarest letters in English, which was ultimately found by Darryl Francis in Xiq-Xhafej (populated place, OSAL) and given in the August 1970 Word Ways. Leslie Card and Ross Eckler in the August 1972 Word Ways considered the smaller problem of 3-sets, tackling only what they judged to be the 182 hardest of the 3276 sets, and finding all but 20, which were filled in in the May 1982 Word Ways. During 4-set work, the other 3094 were all found in NI words, the last to fall being GXX sextuplexing and WWZ zwetschenwasser.

In 1970 l decided to try the general 4-set case. It fit my criteria for a good logological project: it had a clearly delimited goal, but one too distant to be reached in one burst of action, so it could be pecked away at for years. It forced research beyond the bounds of Nl, and could serve as a setting for many of the odd words l came across in my logological reading. And the chance of ultimate success was far greater than that of my earlier long-term project, finding words containing each of the 17576 trigrams from AAA to ZZZ. (On the other hand, it was far less than the chance for a related project: finding words containing all 17576

ordered 3-sets with separation allowed, so that xenon contains XNN but not NXN or NNX. This has never been tried but would be an interesting follow-up to the 4-set project.) I wrote out all 23571 sets in four notebooks and began filling slots from memory and with lists of 4- and 5-letter NI words. Unfortunately, the drudgery outweighed the pleasure, and the prospect of checking longer words later was daunting: ambidextrously alone contains 1001 different sets. I soon put the notebooks aside.

In 1981 Alan Frank visited me and I mentioned the project. He had a database of all words from the Official Scrabble Players Dictionary; turning a computer loose on it, he got about 21500 sets in short order. Then, working himself on missing sets from the 7315 not containing the hard letters J,K,Q,V,W,X,Z, he soon had all but seven. I was able to find BFHH, FFHM, BFHP, FMPP, CFYY and FFYY - can you? (See Answers and Solutions). This left only YYYY unsolved in NI. (We had fyyryryn from O, polysyndactyly from D25, and others; he happened on dacryocystosyringotomy some months later.) With a computer to do the scutwork of extracting sets from words and updating the list, the project could now move into high gear.

Alan told me why the project appealed to him: "It's somewhat mindless at times when I want it to be, but I can think about it profitably if I want to do that. For example, thinking of possible places to find UUVV, and finally coming up with 'vulture-wise'. It's a good computer project. If I find a word with BBHWXX, the computer will tell me if that improves any sets. Theoretically, all sets are findable with normal English words; it's only happen-stance that ex-Jacquardizer is not in the dictionary, whereas the fundamental structure of English assures having to go to outrageous lengths to find the trigram QPX. I like looking through dictionaries, in moderation. I like working on projects with other people; usually better-executed than ones done alone, and more fun."

The programs, for simplicity, accepted only all-upper-case letter input. Once we went beyond Scrabble-legal Nl words, it became necessary to introduce tags to indicate orthographic matters like capitalization and hyphens. Since such factors made a word less desirable, we adopted the convention that any word was superior to any other word with a longer tag and should replace it in the database. Extending this idea, since we wanted to keep as close as possible to NI words, abbreviations for sources of non-NI words were incorporated into the tags. Sources having less satisfactory word-stocks would get longer abbreviations, so words from them would have longer tags and be apter to be replaced. Thus all general dictionaries had 1-letter codes, while non-reference works (for example, newspapers and textbooks) had codes of 4 or more letters. Later, secondary tags were introduced to handle diacritics and other information that did not affect the acceptability of a word. Some sample entries, with + separating the word and its tags: BFKQ BREAKFASTEQUIPAGE+O-+@EQ can be decoded as breakfast-equipage,

tag length 2, in the Oxford English Dictionary, found under "equipage". CFXZ CAZAUXFRECHET+OSFR\*-\*+' is Cazaux-Frechét, with tag length 7, in the Official Standard Names Gazetteer for France, two capital letters, an acute accent that is not considered part of the E and does not lengthen the tag. JWWZ WOJCIESZOW+NIA\*+' is Wojcieszów, tag length 4, in the Rand-McNally New International Atlas, not usable as a JOOZ set because the accent on the second O makes it a different letter in Polish (as do umlauts in German or Hungarian); the program treats lower-case letters as non-letters, so it will not try to put this word in the JOOZ slot. We defined the tag of a 4-set as the tag of the best word known to contain that set.

For the most part, we did not try to distinguish levels of acceptibility within sources. Thus rare and obsolete N12 words can supersede everyday, but longer, words, and if a name is transcribed three different ways in M79, all three have equal status. However, we added a few tag suffixes in special cases. OBS, for example, is used for words showing obsolete spelling conventions (mainly in O), DEF for words found only in definitions of other words, and 2 for words from 2-word phrases. Examples are MQVW vmqwhyle+OOBS+@umquhile, FKPV fun-provoking+DEF+@crazy house (in NI3), and CJMX Jarisch-Herxheimer+2.

Such a crude and one-dimensional measure of acceptability is of course open to objections. Why, for instance, should N12 and N13 words be superior to those of other dictionaries, unabridged (like C,F and O) or not? Basically, for expedience; to avoid cluttering the database and its printout with tags on every word, we say that any untagged word is to be found in one of a specified set of references. N12 and N13 are a natural set. They are widely available, have a huge word-stock, are few in number (so that untagged words are little trouble to locate), and avoid the weird obsolete spellings of O. Giving preference to N1 reduces the number of tags needed. As another example, is UWWW chuck-will's-widow, with its +-'- tag, really on a par with a medical-dictionary term like BBXX hexabromdioxyphenylcarbinmol+D23? Probably not, but there would likely be differences of opinion among logologists as to which is worse.

Crude or not, the system generally works satisfactorily, and anomalies can be handled individually. For example, of the 1997 sets obtainable from pneumonoultramicroscopicsilicovolcanokoniosis, shorter untagged words were known for all but KMPV. We nonetheless used Pervomaisk for KMPV; though capitalized, it was much shorter and less artificial. Alan later eliminated the problem by finding a word better than either; can you think of uncapitalized 10- and 11-letter KMPV words from NI2? (If stumped, see Answers and Solutions.) And for VVVV, we prefer nivvi-nivvi-nak-nak+WD to the shorter Vivstavarv+OSSW, though both have tag length 5, because the former is more English and three hyphens aren't that much worse than one.

For really far-out words, intuition breaks down completely. How

should one rank strange coinages from Rabelais, Joyce, and the Guinness Book of World Records (or, worse yet, logologists); esoteric chemical terms full of digits and parentheses; phantom words that slipped into dictionaries as errors or hoaxes; or pre-English Anglo-Saxon words? For the most part, we just assign them whatever very long tag seems fit at the moment. We regard all words with tags above length 10 or so as terrible, and don't worry much about what supersedes what.

Even for less esoteric words, deciding the level of acceptability of words is a problem in any wide-ranging logological project. The problem was greater here because we had to assign exact numerical values. For example, -itides is explicitly given as a plural of the suffix -itis only in N13 and perijejunitis is only in N12, so is perijejunitides acceptable? We decided it was, without even a tag. NI2 gives no plural for jack-in-a-box; does that mean the plural is the regular, but odd-looking jack-in-a-boxes, or did they just lack information? O explicitly shows jacks-in-boxes, so we let that go as untagged NI2 (not O), with some misgivings on my part. The notes under number in NI3 indicate that fractions can be written as single hyphenated words if used as adjectives, but does this apply if numerator or denominator is itself hyphenated? I suspect not, and have argued for a long disapproving tag (FRAC-TION) for semi-sanctioned coinages like VWWX twenty-six-twentyseventh. I've argued that we can't make inferences from one compound to another. Fifteen-hundred-word in A Manual of Style does justify WXXX six-hundred-sixty-six-word. Six-quarter-cattle in O does not show that CQWX six-quarter-cow exists. Alan proposes that anything with a (subjectively estimated) probability of 5 per cent of ever having occurred in print is includable, with an appropriately long tag. 1 disagree.

On the other hand, I pushed to have the 585-letter surname Wolf-eschlegl... from <u>Guinness</u> included, though it's clearly a logologically-motivated coinage. It's in a common modern reference, and the name of a real person is more legitimate, less easily foisted upon the world, than any purely literary coinage. And I'm not strongly against pluralized proper names. Even if there is only one place in China named Jiangjunqiao, JJQS Jiangjunqiaos+NIAPL could mean "places like Jiangjunqiao." A tag suffix like PL would be in order, though.

Like our tagging principles, our word-hunting strategies differ. Alan concentrates on coining plausible words containing needed sets and trying to find them, in one case spending several hours in the library on an unsuccessful search for extremely-low-frequency. I'm more apt to plow word by word through a reference or, if pickings are slim, through the most promising parts, like exj-, q-, sq- and x- words. This is far slower, but surer. For example, we both did a lot of coining in trying to find an NI word for GKTX: ex-knight, ticket-fixing, textbooking, knitting-box, tackle-box, etc. We found extinguisherlike+O, mixing-stack+F, and striking-box+O, but nothing in NI until an exhaustive search of NI3 turned up alkoxylating. The search for GMXX maxixeing was

likewise fruitless; unlike so many other dances, this was never a verb in any reference checked. Now VWWW is proving intractable. Willow-woven, widow-wives, window-swivel and other compounds are not to be found. View window is a two-word term in NI3, so perhaps it can be found hyphenated somewhere, but ultimate success seems as likely to come from some mindless search.

As the project advanced, the data-processing and output-generating programs have grown more numerous and complex, deserving an article of their own. Alan will use them as examples in an article on computer logology.

With this background, we can now turn to some results. As mentioned above, the 7315 non-JKQVWXZ sets were completely solved early on. This simplified searches, since words not containing one of the seven "hard letters" could be ignored (except when trying to shorten an already-found set). Then, one by one, other letters were removed from the hard-letter list, adding to what could be passed over. First were sets with K, an additional 1570 for a total of 8885. For four of them, only tagged words are known: BFKK knife-backed (flanker back is two words in N13 addenda), FKKK Kirkfieldbank+TIG, GKKK kirk-skailing+O, and KKKP Pakokku. Next was W, 16 tagged in 10626 sets; the 12 new tagged sets are BBWW wibbly-wobbly, BCWW twice-blowing, BPWW whippoorwill's-boots, yellow-brown, CWWW twice-widowed, FWWW widow-sawfly+O, KWWW swallow-woodpecker+O, UUUW Shukulumbwe, UUWW wau-wau, UWWW chuck-will's-widow, WWWW wow-wow, and WYYY y-wryly. was more intractable; adding Z sets gives us a total of 12560, more than half of all sets, but 85 of them are tagged. This is so many that I will only list the 14 of the 69 added ones with tags longer than 1: BBFZ Fitzgibbon+F, BFKZ furze-break+O, BFWZ BWWZ zebra-swallowtail+F, FFFZ Fitzjeffrey+NUCL, zebra-wolf+0, FFWZ zwolffingerdarm+D24, FFZZ frizzle-frize+O, FMMZ Fitzsimmons+A, FMWZ Fitzwilliam+F, FPWZ zephyr-flower+F, FWWZ Safarewiczowa+M80, PWWZ and WWWZ powwow-wizard+O, and UWWZ Wurgwitz+NIA. One could remember the needed K and W sets, but the Z sets are too numerous, and it remains necessary to keep a list for checking when a likely word with Z is found. This is even more true of the V sets; adding them brings us to 14950 solved sets (62.94% of the total), including 20 new ones with tags longer than 2: BBVZ Bezzubov+M79, BVVV Novobarvinovka+OSNU, BVVW Traveling-wavetube+EDS, BVWZ Bovenwezet+OSBE, FFKV Fefelovka+OSNU, FKVV Varfolomejevka+NIA, FVVZ Voelfling-les-Bouzonville+OSFR, FVWZ Menzenschwand-Vorderdorf+OSWG, HVVV Verchovcevo+NIA, HVWZ Hazelnutweevil+C, KVVV and VVVZ lvanovo-Voznesensk, KVWZ and VWYZ skywave-synchronization+EDS2, MVVZ Adzvavom+NIA', PVWZ Onze-Lieve-Vrouwpolder+OSBE, VVVV nivvi-nivvi-nak-nak+WD, VVWZ Wez-Velvain +OSBE, VWWW twenty-two-twenty-sevenths+FRACTION, and VWZZ Zwevezele+NIA. Some might say that VWWW is still a gap. Two more letters are close to being eliminated: only FKXX, VWXZ, VXXZ and WXXZ remain for X, and FJJW, FJJY, FJJZ, JJJW and JJVW for J. Q, however, is still far off; even only considering non-IX sets with only one Q, there are still twelve missing: BFQW, BQVW, FFQZ,

FQVV, FQVZ, FQWZ, KKQZ, KQWZ, PQWZ, QVVW, QVVZ and QVWW. Can readers help us to improve the tagged sets above? It's possible; UUUW, for example, has been improved twice lately, from Wulanwusu+NlA and wound-fungus+O.

It seemed to me that the order of elimination might serve as a measure of the "friendliness" of letters, their ability to occur with others in a word. Thus the most unfriendly letters, worst listed last, would be KWZVXJQ, and the hardest repeat-free set to find would not be JQXZ (rarest letters) but JQVX; indeed, this remains a hole. But this is no help in estimating the friendliness of the other 19 letters, so Alan wrote a program that would give a numerical measure of friendliness by using the want list. Whenever a letter occurs in a tagged set, the tag length is added to the score, completely missing sets being assigned an arbitrary tag length of 25. Thus the natural DFKX flex-cracked adds only 1 to the scores of D,F,K and X. We must go much farther afield for VXYZ Croix-lez-Rouveroy+OSBE; since these four are more reluctant to come together, their scores are increased by 8. Note that VVXZ Vieux-Villez+OSFR adds 14 (double the tag length) to V. The results agree well with the earlier measure: Extroverts (83-155) EAISON, Friendly (217-542) LUTRDCG, Neutral (654-1000) HMYPB, Stand-offish (1696-1811) FK, and Hermit (2379-6414) ZVWJXQ. The only large inconsistency is the unfriendliness of W. The reason is that W combines well with the friendly letters, and so is easy to eliminate early, but goes very badly with unfriendly ones, notably Q. A list of 127 tagged or missing sets containing one Q and no other rare letter (JQXZ), shows 67 containing W. (By the way, we do have JQWX Xiwujieqiao+OSNCP.) At the other end of the scale, two letters' ranking do not match their commonness. It's understandable with H, which is common in English primarily due to its appearance in a small number of words, but T is surprising. Labials seem to be generally unfriendly, as the placement of WVFBP shows. The unfriendliest 3-set is not surprisingly JQX; the twenty-six foursets containing it have tag-lengths summing to 357, compared to 277 for the runner-up, JXX. As an indicator of how far the project has progressed, among the 126 sets containing only the six hermit letters, we have something for 97.

At present we have untagged words for 21980 of the 23751 sets, or 92.55%. Adding hyphenated and apostrophized words raises this to 22335, or 94.04%, and capitalized N1 words (including names from the N12 Gazetteer and Biography sections) bring us to 22602, or 95.13%. Unpunctuated uncapitalized words from all general reference works come to 22129 or 93.17%. References other than N1 that contributed the most are: O (195), N1A (167), C (68), S (57), R (41), M70 (39), OSAL (32), HNA1 (32), OSFR (27), and R1NG (25). Other sources include 10 other dictionaries (74 entries), 11 other OSN Gazetteers (76), 7 other Mnn's (19), 4 other atlases and maps (15), 21 other reference works (104), 5 other works of all sorts (22), literary inventions by Calvino, Joyce, Rabelais and others (6 works, 32), and 10 semi-justified coinages and unverified citations.

From another viewpoint, the 1695 tagged sets consist of 676 with tag length one, 297 at two, 102 at three, 284 at four, 138 at five, 32 at six, 90 at seven to nine, and 76 terrible ones at ten and up, plus 76 missing outright.

In conclusion, we would like to make an appeal for help to Word Ways readers. We list below approximately the worst one per cent of the sets, those most in need of improvement. First is a list of 76 sets for which we have nothing at all; then follows a list of 166 sets with tags of length 7 or more, each followed by its tag length. In addition, capitalization shows how badly a set is needed. Tag length 7 sets are printed as xxxx, 8 as xxxX, 9 as xxXX, 10-14 as xXXX, and 15 or more as XXXX.

NI3 has been searched completely by a human, and computer searches have covered most above-the-line terms in NI2. Obvious compounds have been checked in all the major unabridged dictionaries, and letters J,Q,X and Z have been checked in toto in O. Promising territory includes chemical and other technical terms, gazetteers for a few countries not in the Official Standard Names series, like Poland and the Union of South Africa, and articles and books likely to include useful compound words, such as a book on jukeboxes for JX-- words.

When the stream of improvements has largely dried up, we will make copies of the latest printout (60 to 70 pages) with detailed explanations and bibliography, perhaps as a Word Ways monograph.

BFIQ DKQX FIMQ FQVZ JKQW JQQZ JVWX KQXX QQVW QWXX

BJQW F BJQX F BKQQ F BPQQ F BQVW F	FQZ FJ GQQ FJ JJQ FJ JJW FK JJX FF JJY FQ	PQ FQWZ QV JJJW XX JJQW QQ JJVW QQQ JJVW QU JJWX	JQQW JMQX JPXX JPXX JPXX	JQVX JQWW JQWZ	JVXX KKQX KKQX KKQX KKQX KKQX	KQXZ PQQV PQWZ QQQV QQQZ QQRW QQTX	QQWZ QVVW QVVZ QVWW QVWX QVXX QWWX	VWXZ VXXZ WXXZ
bBJQ 14 bBQX 13 bbvX 8 bgjq 7 bjpq 7 bJPX 12 bjqq 7 bpqv 7 bQQQ 14 bQQX 10 bqvx 7 bqvz 7 bqww 7 bGQQ 11 bjkq 7	cFJX cjjQ 8 cjqX cJXZ cKXX cqwx dhjq djqQ dJQX djqz dJXX dQQX dqVX ffqq	8 fFXZ 8 fhjq 12 fJPZ 10 fjqx 13 fjqz 7 FJWW 7 fJWX 8 fkxZ 10 fqqv 7 fqQW 12 fQQX 10 fqwx 9 FQXX	10 fV 7 f2 12 g 7 g 7 g 15 g 12 g 8 g 7 g 9 g 12 h 7 h	/wz 7 /XX 10  KXZ 10  KVX 13  kxx 7  pqq 7  qqQ 8  qqv 7  qvv 8  qvx 7  QXX 11  jjq 7  jqV 8  jqx 7  JWX 12	jjpq 7 jJQT 1 jJQY 1 jjqz 7 jjwY 8	7 j1x 9 jm 8 jM 10 jM 4 jpp 10 jpo 10 jpo 10 jqo 14 jQ 13 jqo 13 jqo 18 jQ	7 7 7 7 9 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9	jrwX 8 jrxx 7 jtxx 7 jUWX12 JVWW15 jVWZ13 jWWX10 jWXX10 jWXY 12 JXXX 16 jXZZ 14 kkwX 8 kKXZ12 knxx 7 kPXX13

kqQQ 9	kXXY 13	ppqQ 8	pwXZ 9	qrwx 7	tvxx 7	vWWW11
kqVV 9	kxXZ 9	pQQY 10	pxxX 8	qrxx 7	txxx 7	vXXX 11
kqvz 7	kxZZ 9	pqvw 7	qQQY 14	qtwx 7	uvwx 7	wWXZ10
krxx 7	lqwx 7	pqwW 8	qQRX 10	QVXZ 16	uwXX 9	wXXX 11
kTXX 13	mQQQ 14	pQWX 12	QQVX 16	qvyz 7	uXXX ll	wXZZ 14
kVWX 13	mqwX 8	pqXX 9	QQVZ 16	QXXX 20	vvvx 7	×XXX 11
kVXX 13	mxxx 7	pvWZ 9	QQWX 15	qXXY 11	vVWX 11	xzZZ 9
kwxx 7	nqqQ 8	pvxX 8	qQXY 10	qXXZ 10	vvwz 7	
kWXZ 10	oWXX 10	pVXZ 10	qQXZ 10	qXZZ 13	vvxz 7	

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C Century Dictionary and Cyclopedia, including Supplements, Cyclopedia of Names, Atlas, 1911

Dxx Dorland's Medical Dictionary, Xxth Edition

EDS Modern Dictionary of Electronics, by Rudolf F. Graf, 1962

F Funk and Wagnalls New Standard Dictionary of the English Language, 1935-53

HNAl Handbook of American Indians North of Mexico, by Frederick Webb Hodge, Smithsonian Institution of American Ethnology Bulletin 30, Washington, US Government Printing Office, 1907

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NIA Rand McNally New International Atlas, date not given NUCL New Union Catalogue of Pre-1956 Imprints, 1968-81

O Oxford English Dictionary, 1971

OSAL Official Standard Names Gazetter for Albania, US Board on Geographic Names, 2nd Edition, 1961

OSBE Official Standard Names Gazetteer for Belgium, 73, March 1963

OSFR Official Standard Names Gazetteer for France, 83. 1964 OSNU Official Standard Names Gazetteer for the USSR, 42, 1970

OSWG Official Standard Names Gazetteer for the German Federal Republic, 47, May 1960

R The Random House Dictionary of the English Language, 1966

R1NG The Ring Index, American Chemical Society, by A. M. Patterson et al., 2nd Edition, 1960

S Supplement to the Oxford English Dictionary, 1972-76-82

TIG Times Index-Gazetteer of the World, 1965

WD Dr. Joseph Wright's Dialect Dictionary, 1898-1905