SCRABBLE®-TILE DOUBLE WORD SQUARES AND RECTANGLES

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What is the largest double word square or rectangle (containing a different English word in each row and column) that can be formed from the 98 non-blank Scrabble tiles? The purpose of this article is explore, and give the answers to, questions like this in which the common word-play form of the word rectangle is constrained by the number of tiles of each letter available in the game of Scrabble, which is:

```
A  B  C  D  E  F  G  H  I  J  K  L  M  N  O  P  Q  R  S  T  U  V  W  X  Y  Z
9 2 2 4 12 2 3 2 9 1 1 4 2 6 8 2 1 6 4 6 4 2 2 1 2 1
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In many logological investigations the question “What should be considered a word?” arises. In this case the answer seems obvious and natural, which is to permit just those words that are legal in the game of Scrabble. For this reason I chose to use the current (6th) version of the Official Tournament and Club Word List (often called TWL) which is based on the Official Scrabble Players Dictionary (OPSD). Readers wishing to duplicate these results or make further investigations can, with a bit of searching, find this word list online (it is usually called TWL66.TXT).

I also decided to require that all the word rectangles be “dense”, in which all of the words in the rows and columns are distinct. It might be guessed that this is not much of a restriction, but there are a surprising number of even large rectangles with repeated words, such as this 7x6 (note the top and bottom row):

```
MADDEST
ARIOETTA
REACHES
ROCKERS
ALIENEE
MADDEST
```

The same rule applies when the rectangle is a square, which in addition to disallowing squares like the rectangle above also prohibits “single” word squares, in which every row word is the same as the corresponding column word.

The search space that must be examined to find all possible Scrabble word rectangles of all possible sizes is, it turns out, small enough to search exhaustively, but only if an efficient search algorithm is used. The primary key to a fast search is to store the word list not as a simple list of words but rather in a data structure known as a trie (from the phrase “fast retrieval”). The time-consuming operation in an exhaustive search is illustrated by the figure below, in which some letters have been filled in already and the question being asked is: which letters of the alphabet are permissible in the place marked “x”?

```
BRIG
AUTO
SX...
```

With words stored in a trie, the legal possibilities for “x” (whose choice produces prefixes RUx and Sx that both occur in the dictionary) can be determined using just a few computer operations, hundreds of times faster than would be required with the dictionary stored as a linear list of words.
Using about 20 hours of time on a home computer I was able to find all word squares and rectangles of all possible sizes and store them in files. The table below shows the exact number of dense Scrabble word rectangles of all possible sizes:

<table>
<thead>
<tr>
<th>Width</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>717</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>22218</td>
<td>927074</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>68622</td>
<td>14592390</td>
<td>63504774</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>81841</td>
<td>21578847</td>
<td>134203047</td>
<td>31214215</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>72093</td>
<td>14317566</td>
<td>39820130</td>
<td>6301875</td>
<td>75087</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>39354</td>
<td>6231340</td>
<td>6792790</td>
<td>304927</td>
<td>1039</td>
<td>1</td>
</tr>
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<td>8</td>
<td>16827</td>
<td>1436057</td>
<td>477441</td>
<td>3938</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>3354</td>
<td>176997</td>
<td>12010</td>
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<td></td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>336</td>
<td>8469</td>
<td>64</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>53</td>
<td>501</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>12</td>
<td>4</td>
<td>11</td>
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<td></td>
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<tr>
<td>13</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summing all the numbers in above table leads gives a grand total of 342,286,026 rectangles. Note that in counting the squares (not the rectangles) one needs to be careful to count only one of each transpose pair.

The table below shows the unique 7x7, the three 8x6's (two of which are very similar), one example each of 9x5, 10x4, 12x3, and the unique 13x2; these have the largest possible width for each height.

<table>
<thead>
<tr>
<th>SNIPPER</th>
<th>AETHEROMA</th>
<th>8x6</th>
<th>AETHEROMA</th>
<th>8x6</th>
<th>FEVERISH</th>
<th>8x6</th>
</tr>
</thead>
<tbody>
<tr>
<td>7x7</td>
<td>8x6</td>
<td></td>
<td>8x6</td>
<td></td>
<td>ITERANCE</td>
<td></td>
</tr>
<tr>
<td>NONHOME</td>
<td>THOLEPIN</td>
<td></td>
<td>THOLEPIN</td>
<td></td>
<td>AERATING</td>
<td></td>
</tr>
<tr>
<td>EIDOLIC</td>
<td>ARRATING</td>
<td></td>
<td>AERATING</td>
<td></td>
<td>BESOTTED</td>
<td></td>
</tr>
<tr>
<td>ESERINE</td>
<td>MISTRACE</td>
<td></td>
<td>MISTRACE</td>
<td></td>
<td>BRIDLING</td>
<td></td>
</tr>
<tr>
<td>ZONATED</td>
<td>ANTEATER</td>
<td></td>
<td>ANTEATER</td>
<td></td>
<td>ENCEINTE</td>
<td></td>
</tr>
<tr>
<td>ZOMETINE</td>
<td>NEEDLERS</td>
<td></td>
<td>NEEDLERS</td>
<td></td>
<td>DEADNESS</td>
<td></td>
</tr>
<tr>
<td>RESECTS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DOGGERELS</td>
<td>NEWFANGLED</td>
<td>10x4</td>
<td>METAETHICAL</td>
<td>12x3</td>
<td>UNMETABOLIZED</td>
<td>13x2</td>
</tr>
<tr>
<td>OVERVALUE</td>
<td>EXHILARATE</td>
<td></td>
<td>AMELIORATIVE</td>
<td></td>
<td>NEURAMINIDASE</td>
<td></td>
</tr>
<tr>
<td>MINIATURE</td>
<td>EPIDEMICAL</td>
<td></td>
<td>GENTLENESS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INUNDATED</td>
<td>MOROSENESS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CEASELESS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The 7x7 answers the question posed in the introduction, as it is the (unique!) double square or rectangle having the largest area (49). In fact, it is not possible to make a square or rectangle larger than this out of the Scrabble tiles even if the two blank tiles are permitted.

Once we have all the double rectangles enumerated and stored on the computer, it is a simple matter to scan through them and find those with various special properties. For example, it is natural to ask which ones have letters that sum to the highest Scrabble score. Here are the top eight:

<table>
<thead>
<tr>
<th>QUESALES</th>
<th>HAPHAZARD</th>
<th>SCOFILAWS</th>
<th>HANADA</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNTIDILY</td>
<td>AQUILEGIA</td>
<td>HYPEREMIA</td>
<td>EXEQUY</td>
</tr>
<tr>
<td>ACHROMIC</td>
<td>SUCCORING</td>
<td>AMATELY</td>
<td>MIZUNA</td>
</tr>
<tr>
<td>FLEABANE</td>
<td>TAEKWONDO (77)</td>
<td>HALAZONES (77)</td>
<td>PLEACH</td>
</tr>
<tr>
<td>FERMENTS (81)</td>
<td></td>
<td></td>
<td>YESSES (76)</td>
</tr>
</tbody>
</table>
SNIPPER       HARIJAN       PRELATE       REPAST
NONHOME       ATOMIZE       SEMILOG       EXARCH
EIDOLIC       WASABIS       HEEZING       DUNKER
ESRINE        EXIGENT       ACTABLE       EVZONE
ZONATED       DYNODES (74)  WHIRLER       FIESTA
EMETINE       SYNDETS (74)  YAREST (73)
RESECTS (75)

The smallest size having a unique highest-scoring grid is 6x2; of the 72,093 6x2’s, only one achieves the maximum score of 37. The most populous size with a unique highest-scoring grid is 4x4; of the 14,592,390 4x4’s, only one achieves the maximum score of 53. These two special grids are shown below.

SHAZAM
HEXADE (57)
CHOW
HOYA
IBEX
COZY (53)

The largest grids containing all one-point Scrabble tiles are 6x6; there are three such:

OSIERS       RUTILE       RUTINS
RENNET       URANIA       URINAL
ANATTO       SINTER       TANTRA
TINIER       INTURN       INTUIT
ETERNE       MARINE       LIERNIE
SIREES       ELATER       EARNER

The largest grid with a perfect checkerboard of vowels and consonants (with Y classified as a vowel) is the 7x6 shown on the left below. Exactly one of the 31 million 5x5’s, the one shown on the right below, has a different remarkable feature: each letter on a black square of the checkerboard has a point value of one, and each letter on a red square has a point value greater than one. This is the largest square or rectangle with this property.

CAPERER
AXONEME
BELABOR
ANIMATO
LITOTES
ACEROSE

SPIVS
HUMIC
AJIVA
GADID
SHEDS

The largest heterogram grids (no letter of the alphabet appearing more than once) are of size 4x4. There are 29 of these, five of which are shown below.

BRIG       CRAP       GYPS       NEWT       TOPH
AUTO       HUGE       LURE       ACHY       IGLU
SLEW       OBIT       ICON       GRIP       CRAB
HYMN       WYNS       BAWD       SUMO       KEYS

The “Roman sum” of a grid is the sum of the values of its Roman numeral letters (M, D, C, L, X, V or I). The five highest-scoring grids in this scheme are shown below.

DELIMIT       CASAVAS       CHIMLA       CHIASM       STAMMELS
ovicide       AMIDINE       LOCOED       HANDLE       CONOIDAL
MICELLAR      MIDDLED       ALIDAD       ALUMIN       ALGICIDE
SLEDGER       ECHELLE       SLEEVE       LORICA       DEALATED
(4410)        LEBERIER       HARMED       LEBRED       (4403)
(4409)        (4407)        ADDERS (4403)
It is not too surprising to find that there are no pangram grids of any size. There are, however, two grids that contain 21 different letters of the alphabet (the record), with just P, V, X, Y and Z missing; they are

    JACKSHAFT
    AQUILEGIA
    MURDERING
    BASSWOODS

and the same grid with the third line changed to MUTTERING.

Define the “diversity” of a word rectangle as (number of different letters) / (total number of letters). The unique grid with the smallest diversity (0.2) is the 5x4 shown below, which only uses the 4 letters A, E, R, S:

    ERASE
    RARES
    AREBAE
    SEARS

There are no grids of any size containing all four of the rare letters J, Q, X, Z, but there are many containing three of the four; the largest is this 7x5 grid:

    QUIVERS
    ALLOXAN
    ITEMIZE
    DRAINED
    SALTERS

The unique grid having the largest number of palindromes (eight, all in the columns) is this 9x3:

    ANTEDATES
    GUAYABERA
    ANTEDATED

The longest palindrome that occurs in any square or rectangle is 7 letters long. Amazingly, there is a grid containing two such palindromes:

    HALALAH
    ALAMEDA
    REVIVER
    PRECEPT
    STREETS

This 9x5 is the unique grid with the largest number of distinct words contained in the rows and columns (reading only right or down is allowed, not left or up):

    TABULATED
    ABORIGINE
    PARAMENTA
    ESTRANGER
    RESISTERS

Its rows and columns contain these 107 Scrabble-legal words:
If we allow words in all eight orthogonal and diagonal directions, as in a word search puzzle, then the record grid is this 7x6:

```
  STALING  
  HOMAGER  
  ALUMNAE  
  RESPITE  
  IDEATED  
  FODDERS  
```

which contains 161 words:

```
> STALING  TA TALI A AL LI LIN LING I IN HO HOMAGE HOMAGER O OM MA MAG MAGE AG AGE AGER ER ALUM ALUMNA ALUMNAE LUM UM NA NAE AF RE RES RESPITE ES SPIT SPITE PI PIT IT ID IDEA IDEATE IDEATED DE EAT AT ATE TED ED FODDER FODDERS OD ODD ODDER ERS
< NIL LA LAT LATS REQ GAM AM MO OH AN MU ET TI TIP TIPS SER ETA TAE RED REDD EDDO DO OF
Ψ SH SHA SHARIF HA AR RIF IF TO TOLE TOLED TOLEDO OLE LED AMU AMUS AMUSE AMUSED MUSE MUSED USE USED LAM LAMP LAMPAD AMP PA PAD AD IGNITE NIT NITE NE NEAT NEATER EATER GREY GREED GREEDS REE REED REEDS EDS
∧ FIR RAH AH AHS ODE DEL EL LO LOT SUM DAP TIN TING RET EN DEE DEER ERG
⟨ SO SOU SOUP AA MM ANT GAE UP MI SAE
Æ OE NEG
Æ SEI OI AS GS
```

What is the largest grid that contains at least one full-length word on some line going diagonally down and to the right? The answer is 9x5, and there are two such grids, shown below with the diagonal words (which just happen to be in the same place in both grids) underlined. The largest square of this kind is 6x6 and there are 128 of these, one of which is displayed below.

```
<table>
<thead>
<tr>
<th>NANOGRAMS (9x5)</th>
<th>NAPHTHOLS</th>
<th>GAUGED (6x6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OVERLADEN</td>
<td>ANAEROBIA</td>
<td>ARREAR</td>
</tr>
<tr>
<td>MEDIATORY</td>
<td>MICROGLAN</td>
<td>ORANJY</td>
</tr>
<tr>
<td>ORDERABLE</td>
<td>ELEOPTENE</td>
<td>LITTLE</td>
</tr>
<tr>
<td>STYLELESS</td>
<td>REDNESSES</td>
<td>EVILER</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RÆCESS</td>
</tr>
</tbody>
</table>
```