SCRABBLE®-TILE DOUBLE WORD SQUARES AND RECTANGLES

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What is the largest double word square or rectangle (containing a different English word in each row and column) that can be formed from the 98 non-blank Scrabble tiles? The purpose of this article is explore, and give the answers to, questions like this in which the common word-play form of the word rectangle is constrained by the number of tiles of each letter available in the game of Scrabble, which is:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
9	2	2	4	12	2	3	2	9	1	1	4	2	6	8	2	1	6	4	6	4	2	2	1	2	1

In many logological investigations the question "What should be considered a word?" arises. In this case the answer seems obvious and natural, which is to permit just those words that are legal in the game of Scrabble. For this reason I chose to use the current (6th) version of the Official Tournament and Club Word List (often called TWL) which is based on the Official Scrabble Players Dictionary (OPSD). Readers wishing to duplicate these results or make further investigations can, with a bit of searching, find this word list online (it is usually called TWL06.TXT).

I also decided to require that all the word rectangles be "dense", in which all of the words in the rows and columns are distinct. It might be guessed that this is not much of a restriction, but there are a surprising number of even large rectangles with repeated words, such as this 7x6 (note the top and bottom row):

```
MADDEST
ARIETTA
REACHES
ROCKERS
ALIENEE
MADDEST
```

The same rule applies when the rectangle is a square, which in addition to disallowing squares like the rectangle above also prohibits "single" word squares, in which every row word is the same as the corresponding column word.

The search space that much be examined to find all possible Scrabble word rectangles of all possible sizes is, it turns out, small enough to search exhaustively, but only if an efficient search algorithm is used. The primary key to a fast search is to store the word list not as a simple list of words but rather in a data structure known as a *trie* (from the phrase "fast re*trie*val"). The time-consuming operation in an exhaustive search is illustrated by the figure below, in which some letters have been filled in already and the question being asked is: which letters of the alphabet are permissible in the place marked "x"?

```
BRIG
AUTO
Sx..
....
```

With words stored in a trie, the legal possibilities for "x" (whose choice produces prefixes RUx and Sx that both occur in the dictionary) can be determined using just a few computer operations, hundreds of times faster than would be required with the dictionary stored as a linear list of words.

Using about 20 hours of time on a home computer I was able to find all word squares and rectangles of all possible sizes and store them in files. The table below shows the exact number of dense Scrabble word rectangles of all possible sizes:

	Rectangle Height					
Width	2	3	4	5	6	7
2	717					
3	22218	927074				
4	68622	14592390	63504774			
5	81841	21578847	134203047	31214215		
6	72093	14317566	39820130	6301875	75087	
7	39354	6231340	6792790	304927	1039	1
8	16827	1436057	477441	3938	3	
9	3354	176997	12010	13		
10	336	8469	64			
11	53	501				
12	4	11				
13	1					

Summing all the numbers in above table leads gives a grand total of 342,286,026 rectangles. Note that in counting the squares (not the rectangles) one needs to be careful to count only one of each transpose pair.

The table below shows the unique 7x7, the three 8x6's (two of which are very similar), one example each of 9x5, 10x4, 12x3, and the unique 13x2; these have the largest possible width for each height.

```
SNIPPER  7x7        ATHEROMA  8x6       ATHEROMA  8x6       FEVERISH  8x6
NONHOME             THOLEPIN            THOLEPIN            ITERANCE
EIDOLIC             AERATING            AERATING            BESOTTED
ESERINE             MISTRACE            MISTRACE            BRIDLING
ZONATED             ANTEATER            ANTEATER            ENCEINTE
EMETINE             NEEDLERS            NEEDLESS            DEADNESS
RESECTS

DOGGERELS  9x5      NEWFANGLED  10x4    METATHETICAL  12x3  UNMETABOLIZED  13x2
OVERVALUE           EXHILARATE          AMELIORATIVE        NEURAMINIDASE
MINIATURE           EPIDEMICAL          GENTLENESSES
INUNDATED           MOROSENESS
CEASELESS
```

The 7x7 answers the question posed in the introduction, as it is the (unique!) double square or rectangle having the largest area (49). In fact, it is not possible to make a square or rectangle larger than this out of the Scrabble tiles even if the two blank tiles are permitted.

Once we have all the double rectangles enumerated and stored on the computer, it is a simple matter to scan through them and find those with various special properties. For example, it is natural to ask which ones have letters that sum to the highest Scrabble score. Here are the top eight:

```
QUEZALES          HAPHAZARD          SCOFFLAWS          HAMADA
UNTIDILY          AQUILEGIA          HYPEREMIA          EXEQUY
ACHROMIC          SUCCORING          AMATIVELY          MIZUNA
FLEABANE          TAEKWONDO  (77)    HALAZONES  (77)    PLEACH
FERMENTS  (81)                                          YESSES  (76)
```

```
SNIPPER          HARIJAN          PRELATE          REPAST
NONHOME          ATOMIZE          SEMILOG          EXARCH
EIDOLIC          WASABIS          HEEZING          DUNKER
ESERINE          EXIGENT          ACTABLE          EVZONE
ZONATED          DYNODES (74)     WHIRLER          FIESTA
EMETINE                           SYNDETS (74)     YAREST (73)
RESECTS (75)
```

The smallest size having a unique highest-scoring grid is 6x2; of the 72,093 6x2's, only one achieves the maximum score of 37. The most populous size with a unique highest-scoring grid is 4x4; of the 14,592,390 4x4's, only one achieves the maximum score of 53. These two special grids are shown below.

```
SHAZAM           CHOW
HEXADE (37)      HOYA
                 IBEX
                 COZY (53)
```

The largest grids containing all one-point Scrabble tiles are 6x6; there are three such:

```
OSIERS           RUTILE           RUTINS
RENNET           URANIA           URINAL
ANATTO           SINTER           TANTRA
TINIER           INTURN           INTUIT
ETERNE           NARINE           LIERNE
SIREES           ELATER           EARNER
```

The largest grid with a perfect checkerboard of vowels and consonants (with Y classified as a vowel) is the 7x6 shown on the left below. Exactly one of the 31 million 5x5's, the one shown on the right below, has a different remarkable feature: each letter on a black square of the checkerboard has a point value of one, and each letter on a red square has a point value greater than one. This is the largest square or rectangle with this property.

```
CAPERER                          SPIVS
AXONEME                          HUMIC
BELABOR                          AJIVA
ANIMATO                          GADID
LITOTES                          SHEDS
ACEROSE
```

The largest heterogram grids (no letter of the alphabet appearing more than once) are of size 4x4. There are 29 of these, five of which are shown below.

```
BRIG     CRAP     GYPS     NEWT     TOPH
AUTO     HUGE     LURE     ACHY     IGLU
SLEW     OBIT     ICON     GRIP     CRAB
HYMN     WYNS     BAWD     SUMO     KEYS
```

The "Roman sum" of a grid is the sum of the values of its Roman numeral letters (M, D, C, L, X, V or I). The five highest-scoring grids in this scheme are shown below.

```
DELIMIT      CASAVAS      CHIMLA      CHIASM          STAMMELS
OVICIDE      AMIDINE      LOCOED      HANDLE          CONOIDAL
MICELLA      MIDDLED      ALIDAD      ALUMIN          ALGICIDE
SLEDDER      ECHELLE      SLEEVE      LORICA          DEALATED
  (4410)     LEERIER      HARMED      LEERED            (4403)
               (4409)       (4407)    ADDERS (4403)
```

It is not too surprising to find that there are no pangram grids of any size. There are, however, two grids that contain 21 different letters of the alphabet (the record), with just P, V, X, Y and Z missing; they are

```
JACKSHAFT
AQUILEGIA
MURDERING
BASSWOODS
```

and the same grid with the third line changed to MUTTERING.

Define the "diversity" of a word rectangle as (number of different letters) / (total number of letters). The unique grid with the smallest diversity (0.2) is the 5x4 shown below, which only uses the 4 letters A, E, R, S:

```
ERASE
RARES
AREAE
SEARS
```

There are no grids of any size containing all four of the rare letters J, Q, X, Z, but there are many containing three of the four; the largest is this 7x5 grid:

```
QUIVERS
ALLOXAN
ITEMIZE
DRAINED
SALTERS
```

The unique grid having the largest number of palindromes (eight, all in the columns) is this 9x3:

```
ANTEDATES
GUAYABERA
ANTEDATED
```

The longest palindrome that occurs in any square or rectangle is 7 letters long. Amazingly, there is a grid containing *two* such palindromes:

```
HALALAH
ALAMEDA
REVIVER
PRECEPT
STREETS
```

This 9x5 is the unique grid with the largest number of distinct words contained in the rows and columns (reading only right or down is allowed, not left or up):

```
TABULATED
ABORIGINE
PARAMENTA
ESTRANGER
RESISTERS
```

Its rows and columns contain these 107 Scrabble-legal words:

row 1	TA TAB TABU TABULATE TABULATED A AB LA LAT LATE LATED AT ATE TED ED
row 2	ABO ABORIGINE BO O OR ORIGIN RIG I GIN IN NE
row 3	PA PAR PARA PARAMENT PARAMENTA AR ARAME RAM RAMEN RAMENTA AM AMEN AMENT ME MEN MENTA EN
row 4	ES ESTRANGE ESTRANGER STRANG STRANGE STRANGER RAN RANG RANGE RANGER AN ANGER ER
row 5	RE RES RESIST RESISTER RESISTERS SI SIS SISTER SISTERS IS ERS
col 1	TAP TAPE TAPER APE APER PE PER
col 2	ABA ABAS ABASE BA BAS BASE AS
col 3	BORT BORTS ORT ORTS
col 4	URARI
col 5	LI LIMA LIMAS MA MAS
col 6	AG AGE AGENT GEN GENT
col 7	TI TIN TING TINGE
col 8	ENTER
col 9	DE DEAR DEARS EAR EARS ARS

If we allow words in all eight orthogonal and diagonal directions, as in a word search puzzle, then the record grid is this 7x6

STALING
HOMAGER
ALUMNAE
RESPITE
IDEATED
FODDERS

which contains 161 words:

→	STALING TA TALI A AL LI LIN LING I IN HO HOMAGE HOMAGER O OM MA MAG MAGE AG AGE AGER ER ALUM ALUMNA ALUMNAE LUM UM NA NAE AE RE RES RESPITE ES SPIT SPITE PI PIT IT ID IDEA IDEATE IDEATED DE EAT AT ATE TED ED FODDER FODDERS OD ODD ODDER ERS
←	NIL LA LAT LATS REG GAM AM MO OH AN MU ET TI TIP TIPS SER ETA TAE RED REDD EDDO DO OF
↓	SH SHA SHARIF HA AR RIF IF TO TOLE TOLED TOLEDO OLE LED AMU AMUS AMUSE AMUSED MUS MUSE MUSED US USE USED LAM LAMP LAMPAD AMP PA PAD AD IGNITE NIT NITE NE NEAT NEATER EATER GREE GREED GREEDS REE REED REEDS EDS
↑	FIR RAH AH AHS ODE DEL EL LO LOT SUM DAP TIN TING RET EN DEE DEER ERG
↘	SO SOU SOUP AA MM ANT GAE UP MI SAE
↙	GEN RAI RAIA AI PE
↗	OE NEG
↙	SEI OI AS OS

What is the largest grid that contains at least one full-length word on some line going diagonally down and to the right? The answer is 9x5, and there are two such grids, shown below with the diagonal words (which just happen to be in the same place in both grids) underlined. The largest *square* of this kind is 6x6 and there are 128 of these, one of which is displayed below.

NANOGRAMS (9x5)
OVERLADEN
MEDIATORY
ORDERABLE
STYLELESS

NAPHTHOLS
ANAEROBIA
MICROLOAN
ELEOPTENE
REDNESSES

GAUGED (6x6)
ARREAR
ORANGY
LITTLE
EVILER
RECESS