WORD RINGS ON LETTERED FOOTBALLS

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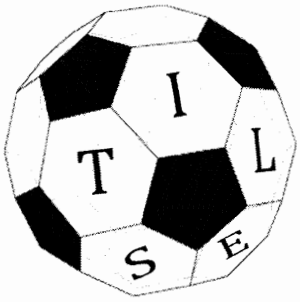
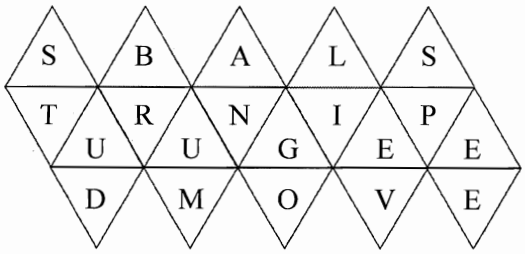


A traditional football (soccer ball) is a combination of 20 pentagons and 12 hexagons arranged in the familiar pattern shown to the right. Suppose a letter of the alphabet is placed on each white hexagon. Can this be done such that the five hexagons that form a ring around each of the twelve black pentagons spell a word when read clockwise (like the tiles labeled TILES in the figure)? The answer is yes, and finding such a labelling makes a nice puzzle - especially if solution by hand, rather than computer, is required.

A number of interesting related questions naturally arise, so to address some of these I wrote a special-purpose computer program that in about nine hours was able to exhaustively enumerate all the essentially distinct solutions (of which there are about half a million) that result when the 8938 five-letter words in the Scrabble® Tournament And Club Word List (6th edition) are used as the pool of valid words. Before discussing some of the more noteworthy arrangements it is necessary, however, to describe the method we will use to display a solution.

We use the fact that a football is a spherical version of the *truncated icosahedron*, in which an icosahedron with 20 triangular faces has its 12 vertices cut off (truncated) to create the pentagonal faces. If we succesively reduce the amount of truncation, the black pentagons become smaller and smaller until in the limit they become the vertices of an icosahedron. This reduction to an icosahedron has no effect on our puzzle, simply changing it to the labelling of an icosahedron such that the five lettered triangles around each vertex spell a word. A solution can be described by labelling a flat icosahedron "net" that can be folded and taped/glued to make a three-dimensional version of the solution.

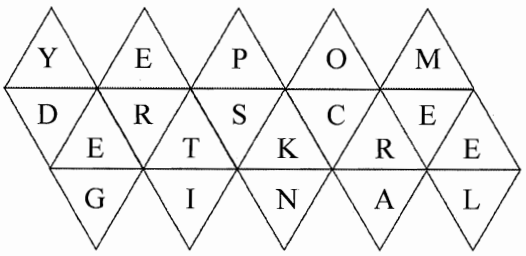
Now, which solutions to our puzzle use the most distinct letters of the alphabet? There are just two that contain 16 different letters, the maximum possible:



Letters: ABDEGILMNOPRSTUV

Words:

SLABS ETUDE BRUTS DURUM URBAN MUNGO
ALIGN VOGIE SPIEL PEEVE STEPS MOVED



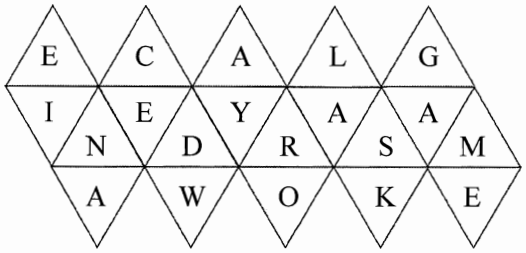
Letters: ACDEGIKLMNOPRSTY

Words:

MOPEY GLEDE REDYE TIGER STREP KNITS
POCKS DRANK COMER LAREE EMYDE ALGIN

Despite the large number of distinct letters, note that both of these solutions have a repeated consonant (S and R, respectively). This suggests the following puzzle: you are given a set of lettered stickers containing just one of each consonant and an unlimited number of vowels (A, E, I, O, U, Y). Is it

possible to make twelve words by putting a sticker on each hexagon? Yes, and the solution with the maximum number of consonants (11) is unique:

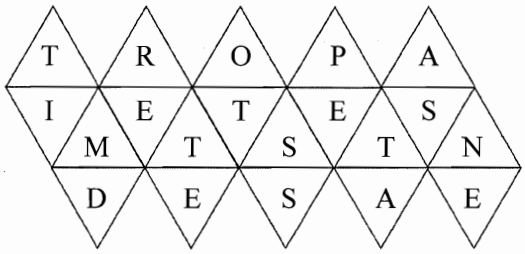


Consonants (each used once): CDGKLMNPRST

Words:

GLACE MINAE NIECE WANED DECAY ROWDY
ALARY KORAS ALGAS SAMEK IMAGE AWOKE

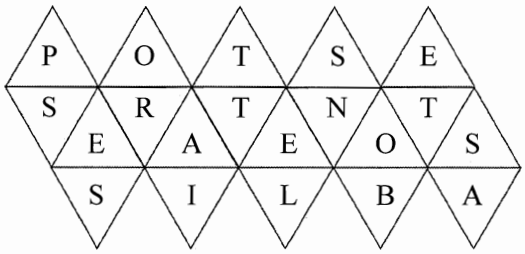
Another question arises from noting that the five letters surrounding a pentagon might spell more than one clockwise word, like TILES and STILE. With a suitable lettering can we create more than twelve words around the black hexagons? Again, the answer is yes - there are a few dozen solutions containing no fewer than 19 distinct words. Of these we can ask, as above, for those that have the most distinct letters. There are just three that attain the maxima of 19 different words and 11 different letters:



Letters: ADEIMNOPRST

APORT DENIM REMIT METED OTTER SETTS
PESTO ASSET PASTE NEATS SATIN AEDES

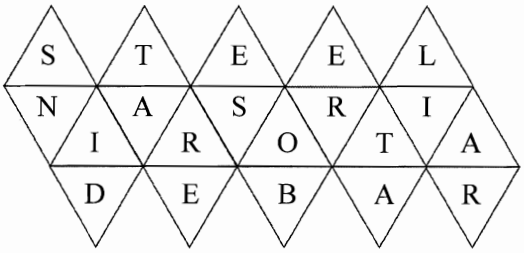
mitre rotte estop stope topes tasse
tepas



Letters: ABEILNOPRST

PESTO ASSES PORES RAISE TAROT TELIA
NETTS NOBLE ONSET SABOT STEPS BASIL

estop stope topes spore ottar seton
serai



Letters: ABDEILNORST

SLEET RANID STAIN REDIA RATES SOBER
EROSE ABORT LITRE TIARA NAILS DEBAR

leets tains tabor relit snail ardeb
barde

In the diagrams above, the lowercase words are those that result from reading one of the uppercase words starting at a different letter.

Thanks to Eric Harshbarger for posing this puzzle.