TRUTHFUL ARITHMETICAL CHARADES

Mike Keith
Furlong, Pennsylvania

In (*WW*, May 2011, p. 99) Anil introduces a construct which I will call a *truthful arithmetical charade*. In the simplest kind, the name for a number (say, FIVE) is converted to its sequence of letter values (6, 9, 22, 5) and then an arithmetic expression (using just the elementary operations of add, subtract, multiply, and divide) is sought that uses these numbers *in order* to produce a value equal to the original number; in this case, FIVE = (-6 + 9 + 22) ÷ 5. Note that parentheses are allowed (to enforce a certain order of evaluation) and the unary minus (as in the "-6" of this example) is permitted.

In (*WW*, Aug 2011, p. 192) Susan Thorpe investigated the multi-part charade, in which the number name is split into two or more parts, with each part having an arithmetic expression that produces the required number. For example,

FI|FTEEN = 6 + 9 = 6 x (20 ÷ 5) + 5 - 14

where the vertical bar indicates the split point and each of the two expressions produces 15. In this article I present some new results for both the single-part and multi-part case.

Single-Word Truthful Arithmetical Charades (no split)

In his article, Anil presented appropriate expressions for the integers 1, 3, 7, 10, 14, 15, 16, 17, 19, and 20. I found these truthful expressions for these additional numbers below 20:

FOUR = 6 - ((15 + 21) ÷ 18)
FIVE = (-6 + 9 + 22) ÷ 5
EIGHT = 5 x ((9 - 7) - (8 ÷ 20))
ELEVEN = 5 + (12 ÷ (5 - 22 + 5 + 14))
TWELVE = 20 - (((23 - 5) x 12) ÷ (22 + 5))
THIRTEEN = (20 + (8 x (9 - 18))) ÷ (20 - 5 - 5 - 14)
EIGHTEEN = -5 + 9 - ((7 x 8) ÷ (20 - 5 - 5 - 14))

and was able to show by exhaustive search that 1, 2, 6, and 9 are not possible. Some of the expressions above are quite tricky and make challenging puzzles. The expression for EIGHT illustrates a particularly subtle feature of some truthful charades: it may be necessary to have non-integer intermediate values during the calculation. One can show by exhaustive search that any truthful expression for EIGHT must utilize fractional intermediate values.

Continuing into the larger numbers, I found a truthful expression for all integers up to 1000 with the exception of SIXTY, which is not possible using just the four standard arithmetic operations. (Note: for numbers greater than one hundred I use the AND-less convention for number names; e.g., 263 is TWO HUNDRED SIXTY THREE) Can the five impossible numbers (1, 2, 6, 9, and 60) be constructed using operations beyond elementary arithmetic? Below are expressions for ONE (due to Anil) and SIXTY using, respectively, exponentiation and a square root. Do expressions like this exist, perhaps using other operators, for 2, 6, or 9?

ONE = $(15 - 14)^5$
SIXTY = $19 - 9 + (\sqrt{(24 - 20)} \times 25)$

I continued searching somewhat beyond 1000 and found that every integer examined had at least one truthful arithmetical expression Is it possible that every integer greater than 1000 does? No, because we know at least one that does not: the number ONETRILLION. The largest possible result that can be achieved comes from multiplying all the letter values: 15 x 14 x 5 x 20 x 18 x 9 x 12 x 12 x 9 x 15 x 14 = 925,888,320,000. But this is less than ONETRILLION = 1,000,000,000,000, so it is impossible to attain the required total. Challenge: find a smaller number between 1000 and one trillion that provably does not have a truthful expression. What is the smallest such number?

Multi-Part Truthful Arithmetical Charades

In the article referenced above, Susan Thorpe gives examples of two-part truthful arithmetical charades for the seven numbers 14, 19, 24, 27, 45, 73, and 78. In total there are 38 numbers less than 100 having two-part truthful expressions, as shown in the table below.

f	THIRT|EEN	13 = ((20 ÷ 8) - 9) x (18 - 20) = (-5 ÷ 5) + 14
a	FOURTEE|N	14 = 6 + 15 + 21 - 18 - 20 + 5 + 5 = 14
	FI|FTEEN	15 = 6 + 9 = 6 x (20 ÷ 5) + 5 - 14
n	SEVEN|TEEN	17 = -19 -5 + 22 + 5 + 14 = ((20 - 5) ÷ 5) + 14
a	NINETE|EN	19 = 14 + 9 - 14 - 5 + 20 - 5 = 5 + 14
	T|WENTY	20 = 20 = (23 - 5 - 14) x (-20 + 25)
	TWENT|YONE	21 = ((-20 + 23) x 5) - 14 + 20 = 25 + 15 - 14 - 5
f	TWEN|TYTWO	22 = -20 + 23 + 5 + 14 = ((-20 ÷ 25) x 20) + 23 + 15
n	TWENTY|THREE	23 = 20 - 23 - 5 - 14 + 20 + 25 = (20 ÷ (8 - 18)) + (5 x 5)
	TWENTYF|OUR	24 = 20 + 23 - (5 x 14) + 20 + 25 + 6 = -15 + 21 + 18
na	TWENTY|SEVEN	27 = -20 + 23 + 5 + 14 - 20 + 25 = 19 - 5 + 22 + 5 - 14
a	TWENTYEIG|HT	28 = 20 + 23 - 5 + 14 - 20 - 25 + 5 + 9 + 7 = 8 + 20
	TWEN|TYNINE	29 = ((-20 + 23) x 5) + 14 = -20 + 25 + 14 - 9 + 19
	THIR|TYFOUR	34 = -20 + (8 x 9) - 18 = (-20 ÷ (25 - 6 - 15)) + 21 + 18
n	THIRTY|FIVE	35 = (20 x 8 x (9 ÷ 18)) - 20 - 25 = (-6 - 9 + 22) x 5
f	THIR|TYSIX	36 = 20 + (8 ÷ (9 ÷ 18)) = (20 x (-25 + 19 + 9)) - 24
	THIR|TYSEVEN	37 = 20 + 8 - 9 + 18 = 20 + 25 - ((19 + 5) ÷ (22 - 5 - 14))
n	THIRTY|EIGHT	38 = 20 - (8 x 9) + (18 x (20 - 25)) = (5 x (9 - 7)) + 8 + 20
n	THIRTY|NINE	39 = ((-20 + 8) x (9 ÷ 18)) + 20 + 25 = 14 + ((-9 + 14) x 5)
	FORTYTH|REE	43 = ((6 - 15) ÷ (18 ÷ 20)) + 25 + 20 + 8 = 18 + (5 x 5)
	FORT|YFIVE	45 = (6 x 15) ÷ (-18 + 20) = 25 - 6 + 9 + 22 - 5
n	FORTY|SEVEN	47 = (6 ÷ (-15 + 18)) + 20 + 25 = (19 x 5) + 22 - (5 x 14)
n	FIFTY|THREE	53 = (6 x 9) - 6 - 20 + 25 = -20 + 8 + ((18 - 5) x 5)
a	FIFTYF|OUR	54 = 6 - 9 + 6 + 20 + 25 + 6 = 15 + 21 + 18
	SIXTYF|IVE	65 = -19 + 9 + 24 + 20 + 25 + 6 = (-9 + 22) x 5
	SEVENTYT|HREE	73 = (19 x 5) - 22 = -5 - 14 + 20 + 25 + 20 + 8 + 18 + (5 ÷ 5)
f	SEVENT|YFIVE	75 = 19 - 5 + 22 + 5 + 14 + 20 = (25 ÷ 6) x (-9 + 22 + 5)
n f	SEVENTY|SEVEN	77 = -19 + (5 x 22) + 5 + 14 + 20 - 25 = ((-19 ÷ 5) + 22) x 5) - 14
n	SEVENTY|EIGHT	78 = (19 x 5) - 22 + 5 = 14 + 20 + (((25 ÷ 5) - 9 + 7) x 8) + 20
	SEVENTYN|INE	79 = 19 + (5 x 22) - 5 + 14 - 20 - 25 - 14 = 9 + (14 x 5)
n	EIGHTY|THREE	83 = (5 x (9 + 7)) + 8 + 20 - 25 = -20 + 8 + (18 x 5) + 5
f	EIGHT|YFIVE	85 = (5 ÷ 9) x (-7 + (8 x 20)) = (25 x 6) + ((9 - 22) x 5)
	EIGHTYS|EVEN	87 = (5 x 9) + (7 x (-8 + 20 - 25 + 19)) = -5 + 22 + (5 x 14)
n	EIGHTY|EIGHT	88 = 5 + ((9 + 7) x 8) - 20 - 25 = -5 + 9 + (7 x (-8 + 20))
	NINETYT|HREE	93 = ((14 + (9 x 14)) ÷ 5) + 20 + 25 + 20 = 8 + (18 x 5) - 5
n	NINETY|FIVE	95 = (14 x (-9 + 14 + 5)) + 20 + 25 = -6 - 9 + (22 x 5)
	NINETYS|EVEN	97 = -14 + (9 x (14 - (5 - (20 ÷ (25 - 19))))) = 5 + 22 + (5 x 14)
n	NINETY|EIGHT	98 = 14 + 9 + (14 x 5) - 20 + 25 = 5 + 9 - (7 x (8 - 20))

The codes in the left column have the following meanings:

- n The split point is in a natural place (between words, or between a word and "teen").
- a The entire charade uses just addition and subtraction (and, as a result, no parentheses).
- f *Any* two-part charade for this number requires non-integer intermediate values.

Note that 27 is both "n" and "a", and is the only such number in the list. The number 39 can be either "n" or "a", but not both. The table above shows the "n" version; the "a" version is

THIRT|YNINE = 20 + 8 + 9 - 18 + 20 = 25 - 14 + 9 - 14 + 5

The calculation shown for a given number in the table above is typically not unique. There may be other expressions that work or other split points that can be used.

Beyond 100, these become even more plentiful. In fact, between 100 and 999 the only numbers *not* possessing a two-part truthful arithmetical charade are 100, 200, 500, 600, 900 and 904. Here, for example, is a natural-split charade for the infamous number 666:

SIXHUNDRED|SIXTYSIX = (19 x 9) + ((24 + 8 - 21) x (14 + 4 + 18 + 5 + 4))
= 19 - 9 + 24 - 20 + 25 + (19 x (9 + 24))

And here is one for a fairly large number:

TWENTY|THOUSAND = 20 x ((23 + 5) ÷ ((14 ÷ 20) ÷ 25))
= 20 x (-8 + ((15 + 21 + 19 + 1) x (14 + 4)))

How about splitting into three parts? Susan Thorpe shows that 28 works:

TWENTY|EIG|HT = ((-20 + 23) x (-5 - 14 + 20)) + 25 = (-5 + 9) x 7 = 8 + 20

and, in fact, this is the only number less than 100 that allows a three-way split. The next numbers having a three-way charade are 103, 108, 110, 111, 113-119, 121-139, 141-149, etc. A larger example is

SEVENTH|OUSAND|TWOHUNDRED = (19 - 5 + 22 - 5 + 14) x (20 x 8)
= (15 + 21) x (19 + 1) x (14 - 4)
= (20 + ((23 x (15 - 8)) - 21) x (14 + 4 + 18 + 5 + 4)

No numbers up to 1000 allow a four-way split, and I did not find any with an inexhaustive search of larger numbers. It seems plausible that there could be some, however, so finding one is left as another challenge for the reader.