INVERTIBLE WORD SQUARES AND RECTANGLES

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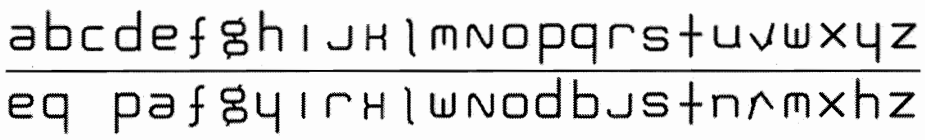
Clap in his walie nieve a blade, he'll make it whissle,
An' legs an' arms an' head will sned, like taps o' thrissle.
– from Robert Burns, "Address to a Haggis"

Eric Harshbarger recently posed an interesting question: what are the largest word squares or rectangles that can be found which transform into another, valid, word square or rectangle when turned upside down? In order to investigate this question, one must decide on an invertible font, in which as many letters of the alphabet as possible also form a letter of the alphabet (ideally a different one, but perhaps the same) when inverted. Combining some of my ideas with some of Eric's, I settled on the font shown below.



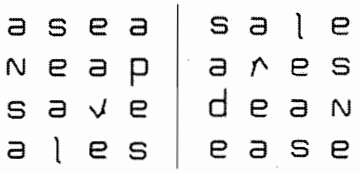
Here each glyph is shown above its upside-down version, and every letter inverts into a valid letter with the exception of "c". Like many ambigrammatic fonts this one is not perfect. The most dodgy letter is the v/r glyph; I decided to allow it because it turns out to be a fruitful one for forming invertible word rectangles, as will become apparent from the lists below. I wasn't completely happy with including some upper-case letters (the invertible N and the K/H pair) in a predominantly lowercase font, but again these turned out to be quite useful.

For the list of valid words I used the latest version of the Scrabble Official Tournament and Club Word List (TWL06). I decided to consider only those squares and rectangles that are *dense*, which means that all N+M words in its NxM grid are different. An invertible NxM grid is said to be *invertibly dense* if the 2N+2M words in the original *and* inverted grids are all different.

A computer program was employed to exhaustively form all possible dense word squares and rectangles of various small sizes and find all the invertible ones. A total of 100 invertible 4x3's were found, of which 41 are invertibly dense, revealing 14 different words when viewed normally and upside-down. Here are two examples:

(In each of our examples the original grid is shown to the left of the vertical rule, its upside-down version to the right.)

Exactly two invertibly dense 4x4's were found, and these have the maximal area (16 units) of all invertible squares and rectangles. These two are the one shown below and the same one with the "l" changed to a "t".



One kind of invertible 4x4 is remarkable even though it's not invertibly dense. These are squares which when inverted turn into their own transpose! There are five of these - the one shown on the left below, and the one on the right with the "l" optionally changed to a "t" and/or the "z" optionally changed to a "t".

pans	pals
anon	ani l
l ine	none
sled	sned

seal	spaz
page	ease
asea	aged
zeds	leas

The first of these owes its existence to the fact that TWL06 includes the Scottish word "sned" (to cut off or prune, as vegetation). See the lines by Robert Burns quoted above for one famous instance of this word.

And finally, here are the three invertibly dense 5x3's (in the one on the left, the "m" can be changed to a "t"):

s + e m s	a l a n e
p a y e e	a a h e d
a n e l e	s w a + s

l e a v e	s + e + s
a a h e d	p a y e e
s + a + s	a ^ e a l