ANSWERING THE SALLOWS CHALLENGE

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In his February 1990 **Word Ways** article, "The New Merology," Lee Sallows challenges the reader to find more than 46 self-descriptive number names between 0 and 99 by assigning suitable integral values to the different letters in each name. It's easy! The key idea is to "square" the array as described in the article "38 Self-Descriptive Number Names" in the February 1990 **Word Ways**.

We attack the problem by first solving a slightly more general one: allow letters to be assigned any real-number values, not merely integers. By so doing, we reduce the problem to one encountered in high-school algebra: the solution of 15 linear equations in 15 unknowns. We then show how the solution can be modified to solve the Sallows challenge.

Ideally, in squaring the array, we would like to make each name in the two sets (ONE, TWO, ... NINE) and (TWENTY, THIRTY,.. NINETY) self-descriptive, so that one could form a total of 9 + 8 + 9x8 = 89 self-descriptive names. However, this is clearly impossible; one can select TY to satisfy only one of the pairs (SIX, SIXTY), (SEVEN, SEVENTY) and (NINE, NINETY). To fix ideas, we select (SEVEN, SEVENTY) and plan to include SIXTY and NINETY as well (but not SIX and NINE) in the self-descriptive set. This will give a total of 7 + 8 + 7x8 = 71 self-descriptive number names. Since there are 15 different letters (E,F,G,H,I,N,O,R,S,T,U,V,W,X,Y) to be assigned numbers, the problem reduces to the solution of 15 linear equations in 15 unknowns. The 71 names can be increased to 74 by selecting a suitable value for Z from ZERO, and for L from ELEVEN and TWELVE.

The solution proceeds as follows: SEVENTY - SEVEN = 63 = [T+Y] and EIGHTY - EIGHT = 72 = Y; hence T = -9. Next solve the three equations O + [N+E] = 1, W + O = 2 - T = 11, and W + [N+E] = 20 - T - [T+Y] = -34 for the unknowns W, O, and [N+E]. U is determined from FORTY - FOUR = 36 = [T+Y] - U. From THIRTY - THREE = 27 = I + 63 - 2E, one obtains I - 2E = -36. This equation, together with N + E = -22 and N + I = 90 - [N+E] - [T+Y] = 49, may be solved for N, E, and I. The solutions for Y,T,W,O,E,I,N and U are given in the (SEVEN, SEVENTY) column in the table on the next page.

The remaining seven letters all have half-integer solutions. F is determined from FIFTY = 50; with F in hand, V is determined from FIVE = 5 and R, from FOUR = 4. Knowing V, S is ascertained from SEVEN = 7, and X in turn from SIXTY = 60. Using R, H is obtained from THREE = 3, and G in turn from EIGHT = 8. The half-integer values of these letters are given in the second column below.

A number of alternative solutions are possible. One can elect to include SIX instead of SIXTY, or NINE instead of NINETY in the above set. (One can even include both SIX and NINE, although this leads to 69 self-descriptive numbers instead of 71.) Or, one can select the pair (SIX, SIXTY) or the pair (NINE, NINETY) as a base for another set. In the first three columns of the table below, we summarize the changes that occur in the assignment of numbers to letters:

	SIX, SIXTY	SEVEN, SEVENTY	NINE, NINETY	SEVEN, SEVENTY (integers)	SEVEN, SEVENTY (integers<100)
Y	72	72	72	72	72
T	-18	-9	9	-9	-9
W	3/2	-12	-39	-12	-12
O	37/2	23	32	23	23
E	98	107	125	107	20
I	169	178	196	178	4
N	-231/2	-129	-156	-129	-42
U	18	27	45	27	27
F	-173/2	-191/2	-227/2	-120	-18
V	-351/2	-369/2	-405/2	-160	-1
R	54	99/2	81/2	74	-28
H	-229	-503/2	-593/2	-276	0
G	-12	-33/2	-51/2	8	-7
S	111	213/2	231/2	82	10
X	-274	-575/2	-665/2	-263	-17
Z	-341/2	-359/2	-395/2	-204	-15
L	8	7/2	-11/2	-21	-6

None of the alternative solutions lead to additional self-descriptive number names. Note that the (SIX, SIXTY) set has three fewer half-integer assignments than the others.

We now turn to Sallows' challenge. Clearly, we cannot make all 15 of the number names self-descriptive, but it is possible to include 14 of them, leaving out only FIFTY. This yields the same solution for Y,T,W,O,E,I,N and U. Pick an integral value for S, X, or V, whereupon the other two letters are determined from equations using SEVEN and SIXTY. Next, select an integral value for either H or R, and the other is determined from an equation using THREE. The remaining two letters, F and G, follow at once from FIVE (or FOUR) and EIGHT. This results in 63 self-descriptive number names, which can be augmented to 66 by adding Z and L as described previously. Results are shown in the fourth column of the table above.

Sallows suggested the additional restriction that the integers assigned to letters all lie within the range of the self-descriptive number names they produce. Obviously this is impossible to satisfy for negative integer assignments, but the absolute values can be restricted in this manner. One can square the sets (ONE, TWO, THREE, FOUR, FIVE, SEVEN, EIGHT) and (TWENTY, THIRTY, FORTY, SIXTY, SEVENTY, EIGHTY) to produce 55 self-descriptive number names between 0 and 88. As before, this can be augmented to 58 self-descriptive number names with the addition of Z and L. As indicated in the fifth column of the table on the previous page, all integer assignments lie between -42 and +72.

A few other discoveries may be of interest. The first six integers can be made self-descriptive using numerical values that represent every sixth from 1/6 to 13/6:

ONE = 1/3 + 1/2 + 1/6
TWO = 2/3 + 1 + 1/3
THREE = 2/3 + 7/6 + 5/6 + 1/6 + 1/6
FOUR = 4/3 + 1/3 + 3/2 + 5/6
FIVE = 4/3 + 11/6 + 5/3 + 1/6
SIX = 2 + 11/6 + 13/6

If one is interested in "tight" integer sets making ONE through NINE all self-descriptive, consider the first assignment below. The second assignment adds ZERO as a self-descriptive number.

-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
		F	S	H	O	E		W	I	T	N	R	U	G	X	V	
R		G	X	V	O	E		W	I	T	N		U	F	S	H	Z

Finally, if one wishes to maximize the number of consecutive integers that are self-descriptive under integer assignment, we cannot improve on the 31 numbers NINETEEN through FORTY-NINE given by Sallows. However, the corresponding number of consecutive numbers possible under non-integer assignment is 51. Those from TWENTY through SIXTY-NINE can be obtained using the arguments given earlier in this article. SEVENTY cannot be added because TY cannot simultaneously satisfy (SIX, SEVEN, SIXTY, SEVENTY). NINETEEN can be added as follows. Since SIX and SIXTY must both be included in this set, T + Y = 54. From TWENTY - TWO + ONE = 19 = 2[E+N] - 54, one concludes that [E+N] = -35/2; from the equation THIRTY - THREE = 27, one concludes that I - 2E = -27; and from NINE = 9, [N+I] = 9 - (-35/2). Solving these equations for E and N, one obtains E = 71 and N = -178/2. If NINETEEN is to be self-descriptive, TEEN must equal 10 and T must therefore equal -87/2. Other letters are assigned integer values as described previously.