INSERTION-DELETION NETWORKS

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Word Ways has recently published articles based on my computer analysis of Carrollian word ladders and networks. The editor subsequently suggested that 1 might use the computer to extend his study of insertion-deletion (ID) networks in the August 1987 **Word Ways**, to dictionaries larger than the Merriam-Webster Pocket Dictionary. In this pair of articles, I present the results of a few explorations. The ID network is very complicated. Although its analysis based on a larger dictionary is beyond my capability, many interesting things can be found.

In an ID network, a word of n letters is linked to a word of n-1 letters if the first can be converted to the second by the deletion of a single letter, such as COAT to CAT. Beheadments and curtailments are allowed as well. The advantage of an ID network over the standard Carrollian one is that it permits one to construct word ladders with words of different lengths.

My database is the Official Scrabble Players Dictionary (OSPD) augmented in several ways, primarily by the addition of words extracted from the WORDPERFECT speller (WSPS). Such augmentation is necessary because the OSPD includes only words of eight letters or less. The WPSP is about as large as a collegiate dictionary, but, except for legal, medical, and commercial terms, it is only half as inclusive as the OSPD. Most of the augmentation consists of 9-letter words; OSPD yielded about 13,000 derivatives of 8-letter words, to which I added some 10,300 from WPSP. Some of the findings given here are based on the OSPD alone; others are based on the augmented list.

Successive Deletions of 9-Letter Words

In order to develop an ID network, one must search in both the insertion and deletion directions. I do not know any practical way to do a computer search in the insertion direction. Therefore, to get started on exploring the network, I developed a process to search in the deletion direction only, constructing lists of D-words.

Start by accepting all 2-letter words from the OSPD. Define a 3D-word as one that can be deleted to one (or more) 2-letter words, a 4D-word as one that can be deleted to a 3D-word (not just any 3-letter word), and so on. Thus, all 9D-words can be successively deleted, one letter at a time, down to one or more 2-letter words.

The table on the next page gives counts of D-words. The basic count consists of OSPD words plus the 10,300 9-letter words from

WPSP described above. To this l added 6-letter through 8-letter WPSP words, as well as a few words of 2 to 5 letters in length from Chambers; this is the count in the table below. A full list of 9D-words is given in the Appendix.

Length	Words	D-Words	Percent
2	98	98	
3	943	744	78.9
4	3786	2676	70.7
5	8568	4713	55.1
6	15035	4675	31.1
7	22928	3139	13.7
8	26700	1288	4.82
9	23295	237	1.08

Contrary to expectation, the addition of the 6-letter through 8-letter WPSP words brought in only a few more 9D-words; most gains in 9D-words were from the introduction of WPSP words at intermediate levels. Since all 2-letter words are interconnected in the 1D network (via 3-letter and 4-letter words), all 9D-words are in the network as well.

Hand search of the OED and Webster's Third brought to light 14 10D-words: CARROUSELS, COLONIZERS, COMPOSTERS, COMPOSITED, CRAUNCHERS, ESCALADERS, MACERATERS, MASCULATED, MASCULATES, SCRAPPLERS, SCRATCHERS, SCRAUNCHES, SCROOPINGS, STRANGLERS. There are even two 11D-words, COMPOSITERS and SCRAUNCHERS. The former pluralizes COMPOSITER and COMPOSTER, obsolete variants of compositor in the OED; the latter creates plural agent nouns for CRAUNCH and SCRAUNCH, found in Webster's Third:

compositers-composters-composers-composes-compose-compos-comps-cops-cop
scraunchers-craunchers-craunches-cranches-ranches-rances-races-race-ace

l give below a sampling of 9D-words, each with a single deletion path. By analogy with transdeletion pyramids, these structures might logically be called deletion pyramids:

upraisers-upraises-upraise-praise-raise-rase-ras-as
preachers-preaches-peaches-peaces-paces-pace-pac-pa
ministers-minister-minster-mister-miser-mise-mis-is
wranglers-wrangler-wangler-angler-anger-ager-age-ae
stampedes-stampede-stamped-tamped-tamed-tame-tae-ae
flossiest-flossies-flosses-losses-loses-lose-ose-os
cocreated-cocreate-ocreate-create-crate-rate-ate-at
roadsters-roadster-roaster-raster-rater-rate-ate-at
bespoused-espoused-spoused-spouse-souse-sous-sou-so
muensters-muenster-munster-muster-muter-mute-mut-mu
laureates-laureate-aureate-aurate-urate-rate-rat-at
strippers-stripper-striper-stripe-tripe-trip-tip-ti
gambolled-gamboled-gambled-ambled-amble-able-ale-ae
sparkling-sparking-sparing-spring-sprig-prig-pig-pi
cleansers-cleanses-cleanse-cleans-clean-clan-can-an
bilanders-bilander-blander-lander-lader-lade-lad-la
islanders-islander-slander-sander-saner-sane-sae-ae

klatsches-klatches-latches-laches-aches-aces-ace-ae
smoothies-smoothes-soothes-soothe-sooth-soot-sot-so
achordate-chordate-chordae-chorda-chord-cord-cod-od
blackings-blacking-lacking-laking-lakin-akin-kin-in
escaladed-escalade-scalade-scalae-scale-sale-ale-ae
soldierly-soldiery-soldier-solder-solde-sold-sol-so
grandeurs-grandeur-grander-grader-grade-grad-gad-ad
corselets-corselet-corslet-corset-corse-core-ore-re
mantelets-mantlets-mantles-mantes-manes-mane-man-an
jatrophic-atrophic-trophic-tropic-topic-topi-top-to
supinated-supinate-spinate-spinae-spina-spin-pin-pi

Tree Branching

Let us now think of successive deletions of D-words as forming trees. In the following two trees based on 6D-words, the number of paths is listed after each word. **Brands** has more branching near the root, but **amides** yields more two-letter words (kinds of fruit?). Note that **me** and **ae**, which might have been possible, do not show. In this section, all words are taken from the OSPD.

AMIDES 74
amide 14 amids 32 amies 11 aides 17
  amid 8   amid 8   amie 3   aide 3
  amie 3   amis 8   amis 8   aids 10
  aide 3   aids 10   ides 4
         mids 6
am,ai,ad,as,mi,id,is,de,es

BRANDS 80
brand 18 bands 20 rands 12 brads 18 brans 12
  band 6 ands 4 rand 4 brad 4 bran 4
  rand 4 band 6 rads 4 bras 4 bras 4
  bran 4 bans 4 ands 4 bads 6 bans 4
  brad 4 bads 6   rads 4
ba,an,ad,as

Here are a couple of trees based on 7D-words; all possible two-letter words that might come from **hearths** or **strains** do, in fact, appear. 137 is the largest number of paths from any 7D-word, but **hearths** produces more two-letter words.

STRAINS 137
strain 24 stains 56 trains 57
  stain 12 stain 12 train 12
  train 12 tains 25 rains 16
         sains 19 tains 25
               trans 4
si,ta,ti,ai,an,as,in,is

HEARTHS 130
hearth 36 hearts 54 earths 4 heaths 36
  heart 14 heart 14 earth 4 heath 18
  earth 4 hears 16   heats 12
  heath 18 harts 12   heths 6
         heats 12
he,ha,er,es,et,eh,ar,at,ah,as

When we go to 8D-words, the picture changes. 8D-words do not make very many branches; some do not branch at all. The 8D-words below have the greatest number of paths to two-letter words.

SPARGERS 284
sparger 35 sparges 94 sparers 95
  sparge 18 spares 45 sparer 17
  sparer 17 sparge 18 spares 45
         parges 26 parers 33
         sarges 5
ae,pa,es,ar

PLASTERS 182
plaster 40 platers 68 pasters 62 lasters 12
  paster 14 plater 20 paters 14 laster 6
  plater 20 paters 14 paster 14 lasers 6
  laster 6 plates 34 pastes 34
pa,ae,as,at,la

Let us now look at connections between the full list of 5-letter

and 6-letter OSPD words. There are no OSPD 6-letter words that can be deleted in six ways, but the following can be deleted in five ways: beasts, brands, chards, charts, clamps, coasts, cramps, skites, spates, spicks, spikes, spines, tramps, yearns. All are D-words; yearns contains 64 different paths to two-letter words.

Five-letter words that can be deleted in five ways to OSPD words include peats, boats, and moats. If one allows words such as sars, tais, taks, bors, and pais (in Chambers, but not in the OSPD), then sears, tains, tasks, boars and pains also allow five deletions. Words such as these were christened charitable words by Dave Silverman.

Cares (scares, chares, cadres, cartes, carets, caress) allows insertion at six different places. The following words can be inserted in five places: haler, hales, lader, laker, lever, liner, liter, pases, poser, poses, raper, tiles. The following five-letter words allow the most insertions regardless of place: ailed 13, owing 13, aster 12, rater 12, ashes 11, awing 11, cares 11, ender 11, ashed 10, avers 10, and pases 10. Cares and pases are the only words on both lists.

Here is a list of four-letter words that allow insertions at five different places. These were christened hospitable words by Dave Silverman. Only one five-letter example is shown for each place. These make fine puzzles.
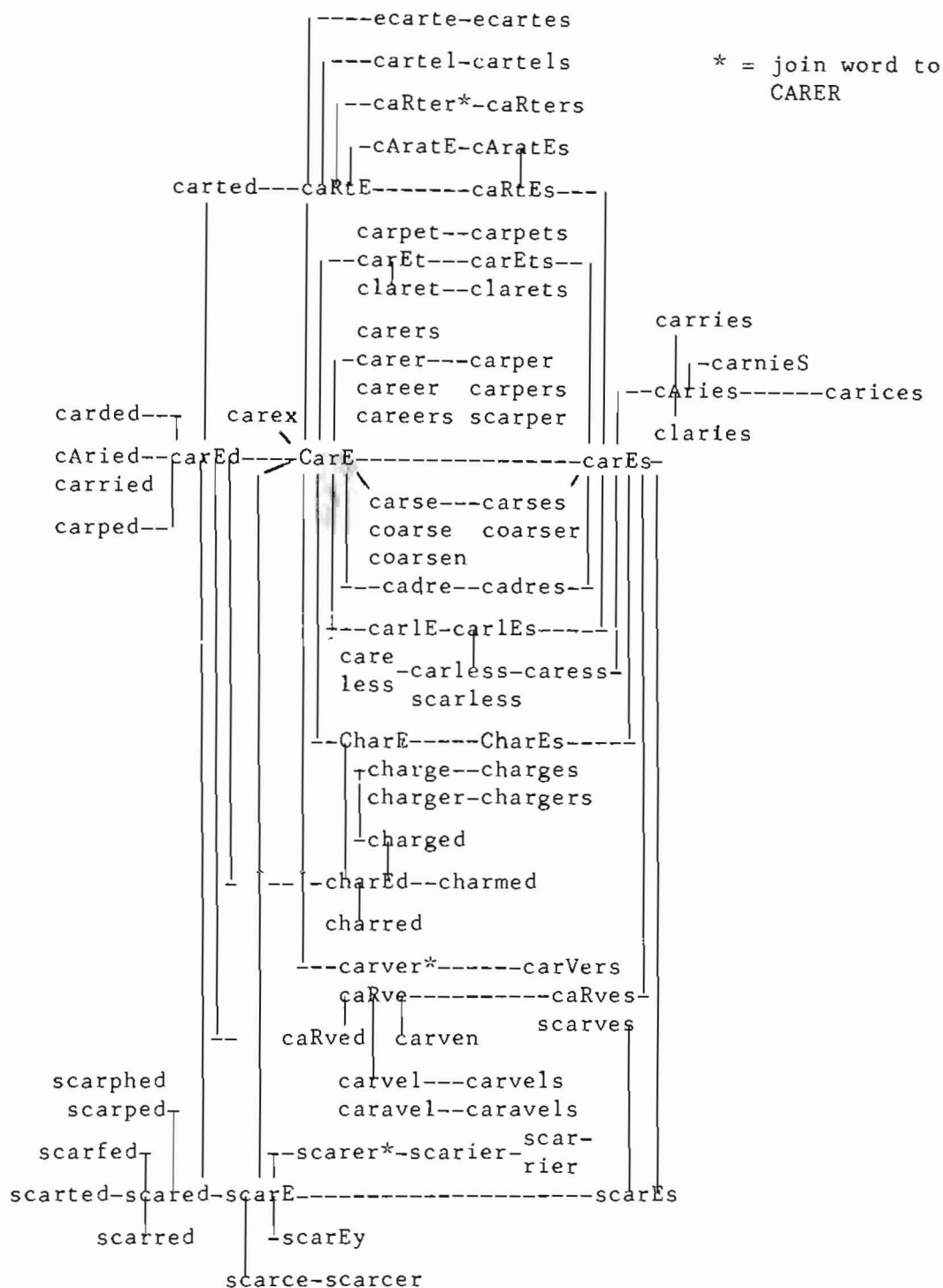
amas: camas, almas, amias, amahs, amass   care: scare, chare, cadre, carve, carex
gees: ogees, ghees, genes, geeks, geese   cans: scans, clans, carns, canes, canso
cops: scops, chops, corps, copes, copse   laps: flaps, leaps, lamps, lapis, lapse
mids: imids, maids, minds, midis, midst   pals: opals, peals, pails, pales, palsy
pans: spans, plans, pains, panes, pansy   pars: spars, pears, pairs, pares, parse
pats: spats, prats, pants, pates, patsy   tars: stars, tsars, tahrs, tarts, tarsi
tile: utile, toile, title, tilde, tiled

Sampling the Insertion Network

Thus far, this article has principally focused on deletions in the 1D network; these were, as previously mentioned, more amenable to computer analysis. On the next page I show a small, but complicated, part of the insertion network: all OSPD words which can be reached by successive insertions from CARE. Partial capitalizations indicate the points joining the rest of the 1D network; for example, CarE indicates that one can delete to CAR and ARE, and cAratE, to CARAT and CRATE. Note that it contains eight 8-letter words: chargers, careless, scarless, caravels, scarphed, scarrier, scarpers, and coarsens. If the OSPD allowed proper names Charles, Clare and Carole could be included; Carole would bring in the additional insertion words caroled, caroler, carolled, caroller, and cariole.

Search Limits

From these studies, I conjecture that if we choose two five-letter or six-letter words at random in the OSPD, there is a fair chance of finding an 1D ladder joining them if the database is at least twice as large as the OSPD. Search would be difficult. However,

135

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                        ----ecarte-ecartes
                        ---cartel-cartels        * = join word to
                        --caRter*-caRters            CARER
                        -cAratE-cAratEs
        carted---caRtE-------caRtEs--
                        carpet--carpets
                        --carEt---carEts--
                        claret--clarets
                        carers                    carries
                        -carer---carper             -carnieS
                        career  carpers       --cAries------carices
carded--    carex       careers scarper            claries
cAried--carEd----CarE----------------carEs-
carried                   carse---carses
carped--                  coarse  coarser
                          coarsen
                        ---cadre--cadres--
                        ---carlE-carlEs----
                        care
                        less-carless-caress-
                          scarless
                        --CharE-----CharEs----
                          charge--charges
                          charger-chargers
                          charged
            -  --  -charEd--charmed
                    charred
                    ---carver*------carVers
                      caRve----------caRves-
                                     scarves
            --  caRved  carven
scarphed              carvel---carvels
scarped               caravel--caravels
scarfed                                scar-
                    --scarer*-scarier-rier
scarted-scared-scarE---------------------scarEs
      scarred       -scarEy
            scarce-scarcer
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finding fully deletable words is easy; there is no reason why it could not be done using an unabridged dictionary.

For those who want to search for 1D ladders or construct networks by hand, there is an excellent computer tool available. Make your word lists into a WORDPERFECT dictionary and use "wild card" look-up. The CARE insertion network was found in this way. Search

for D-words was made by a series of programs written in compiled
BASIC.

APPENDIX

In the following list, capital letters indicate the ways in which
the first letter can be deleted from the 9-letter list of 9D-words.
l guess that there might be 40 or 50 lOD-words to be found. Two
that come to mind at once are sparklings-sparkling-sparing-spring-
sprig-prig-pig-pi and stranglers-strangler-stranger-strange-strang-
stang-tang-tan-ta.

abridgerS	acerBated	acerbateS	Achordate	acierateD	acierateS
ascaridEs	asperserS	bespouseD	bespouseS	bilanderS	blackingS
BleacheRs	boardingS	bowelLing	brAiniest	brashIest	brassiesT
breacheRs	broacheRs	bRushiest	canvaseRs	canvasSes	caresseRs
caroLuses	carouselS	CarouseRS	caRrousel	chaUnters	chordateS
chorusSed	cleanserS	cocreateD	cocreateS	coEmpting	coloniSes
coloniZes	compEting	compLeted	composerS	composSes	composTed
compUting	confinerS	coNsigned	copperaHs	corseletS	coSigning
couplingS	courtIerS	courtLier	craUnches	croupierS	dairyingS
depriverS	diameterS	draggLing	drownDing	duelLists	escaladeD
EscaladeS	espouseRS	Estopping	estrangeD	estrangeS	Estraying
flossiesT	foregoerS	fraggingS	fRillingS	gambolLed	Gestating
gladLiest	glossiesT	gRabblerS	grandeurS	groupingS	GrumblerS
heaLthier	heartiesT	iNsolateD	iNsolateS	IslanderS	Isolating
Jatrophic	klatscheS	LacerateD	lacerateS	laminateD	laminateS
lamisterS	largesseS	launcheRs	laureateD	laureateS	macerateD
macerateR	macerateS	maculateD	maculateS	MalignerS	Maligning
mantElets	manTillas	milliNerS	millinerY	ministerS	muensterS
outrangeD	outrangeS	pauperisM	peArlites	pickeTers	pILasters
piNasters	pinnacleD	pinnacLeS	plaisterS	plankterS	plushIest
PrattlerS	pReachers	prickLier	prissiesT	prompterS	pugGarees
purflingS	rancherOs	reEmergeD	reEmergeS	reflOated	relapserS
rePenters	replaNted	restaRted	Restating	roadsterS	saltinesS
scourGers	scoutHers	scRabbled	scrapingS	ScrapPers	Scrappler
ScrapPing	Scratches	scRooping	shacklerS	Shallowed	SheatherS
shinglerS	shoppingS	skirtingS	smaltineS	smootheNs	sMootherS
sMoothesT	smoothIes	Snappiest	sOlanderS	soldierLy	solVating
sparkLers	sparKLing	sparlingS	spirIting	SplasherS	SPlashier
SpLatters	sPlitterS	sPLitting	splutterS	Sprattled	springerS
springIer	stampedeD	stampedeS	stancheRs	stanchesT	starlingS
Starriest	startLers	starTLing	staUnches	Stoppling	stRanders
strangerS	strangleD	strangLeR	strangleS	StrapperS	Strapping
stReamers	stReamier	Strickled	stRingers	stRingier	stripPers
stRopping	supinateD	supinateS	supplierS	switcheRs	tacklingS
thrasheRs	threaderS	threadIer	tHrillerS	thWacking	trackingS
trampLers	trampLing	tRappings	treadLers	troublerS	truckingS
truckLing	TwaddlerS	TWanglerS	twitcheRs	Twitchier	UpraiserS
Upreached	Upreaches	whittLers	witChiest	wranglerS	wrappingS