# MORE INSERTION-DELETION NETWORKS

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This is the second of two articles that investigate various properties of the insertion-deletion (ID) network. The first article, appearing in the August 1990 **Word Ways**, looked at words fully deletable to 2-letter words, as well as words that can be deleted or inserted in every position (charitable and hospitable words). The complexity of the ID network was hinted at with a depiction of the insertion network of CARE.

This study is based on Official Scrabble Players Dictionary (OSPD) words augmented by words from Chambers Twentieth Century Dictionary. The resulting database has the following properties:

102 2-letter words (86 OSPD) averaging 12.4 joins to 3-letter words

1005 3-letter words (908 OSPD) averaging 6.7 joins to 2-letter and 4-letter words

3800 4-letter words (3688 OSPD) averaging 3.5 joins to 3-letter and 5-letter words

3149 5-letter words averaging 2.5 joins to 4-letter words

A total of 8056 words were used. The 5-letter words were taken from a larger list of 8156 words; only those which join 2 or more 4-letter words were included.

The difficulty of searching for joins in the insertion direction can be avoided by the use of "joiner" tables prepared in preliminary searches in the deletion direction. Subsequent search is very rapid, as is always the case with table look-up, but a large amount of computer memory is required.

## The Span of the ID Network

The span of a network is the number of steps needed to join the two farthest-apart words in the network. To make this more precise, let $n(a,b)$ be the minimum number of steps needed to join words a and b (for example, 6 for SCARFED-scared-cared-care-carte-cate-ATE); then the span is the maximum value of $n(a,b)$, taken over all a and b in the network. In a Carrollian network, wherein all words are of the same size, the span is not difficult to calculate. In an ID network, where we not only have words of different sizes, but size is unlimited, the span must be enormous.

Instead of calculating the span directly, I calculate various conditional spans of the following nature: the span between any two words of the same length n, in a network limited to words of m letters or less (m greater than or equal to n).

Consider first the span of 2-letter words. If one allows only words of three or fewer letters in the network, and restricts oneself to the OSPD, only OF and XU are isolated from the other 84 (OF joins OAF, OFT and OFT, but none of these words allows re-entry to a 2-letter word). Other 2-letter words containing U are rather tenuously connected to the network, through the links -ut-uta-ta, -mu-emu-em- and -mu-amu-am-. The span is 12, as illustrated by UP-ups-us-mus-mu-amu-am-ami-ai-ais-is-ifs-IF. After the augmentation of the OSPD described above, OF remains isolated, but the span of the 101 other 2-letter words is reduced to 8, as illustrated by DA-dad-ad-ado-do-don-on-son-SO (and many other pairs); words solely in Chambers, though few in number, play a very important part in the network structure. If 4-letter words are also allowed, OF joins the network and the span increases to 12: OF-oft-soft-sot-so-soy-oy-boy-by-bay-ay-day-DA. Again, a small change in the conditions has a marked effect upon the span.

Consider now the conditional span between pairs of 3-letter words. Of the 1005 3-letter words, 46 join three 2-letter words, 367 join two 2-letter words, and 390 join only one 2-letter word. 202 3-letter words do not join any 2-letter words; they must enter the network, if at all, via 4-letter words. The following are the most gregarious 3-letter words:

| Word | pas | res | ain | ale | ins | ars | tas | dos | ins | bas | hat | mas | ane | ays | ear | las |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Joins to 2-letter | 2 | 2 | 3 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 2 |
| Joins to 4-letter | 21 | 21 | 19 | 21 | 20 | 20 | 20 | 19 | 19 | 18 | 18 | 18 | 17 | 17 | 16 | 17 |

If the network is limited to words at most 3 letters long, the span is 10 (join two 3-letter words to a pair of 2-letter words with a span of 8, such as DAB to DA and SOU to SO). However, if words of four letters are allowed in the network, then the span increases to 18. Boldface words are found in Chambers but not the OSPD:

VUG-vugh-ugh-pugh-pug-pung-pun-un-mun-maun-man-ma-mae-
ae-**ake**-jake-**jak**-jauk-AUK
VUG-vugh-ugh-pugh-pug-pugs-pus-us-use-user-ser-er-erg-
ergo-ego-sego-**seg**-skeg-KEG
VUG-vugh-ugh-pugh-pug-pugs-pus-opus-ops-opes-pes-apes-
ape-nape-nap-neap-**nep**-neep-NEE

Most of these far-out words are on 4-step peninsulas. If words of five letters or fewer are allowed in the network, more 3-letter words are added to the network, and the span rises to 20: RUC-ruck-truck-tuck-stuck-suck-**suk**-sunk-sun-sung-suing-sing-sin-in-pin-pi-pic-pick-picky-icky-ICY. There are still 50 3-letter words not included in the network. Cig, cry, fez, lym, nth, pyx, sky, tux, veg, vly and zuz are single-word islands, and a number of other 3-letter words form only a group of two with their plurals.

What is the conditional span between 5-letter words? The network is limited to words of five letters as well, since this is the full extent of the database. The span is 26:
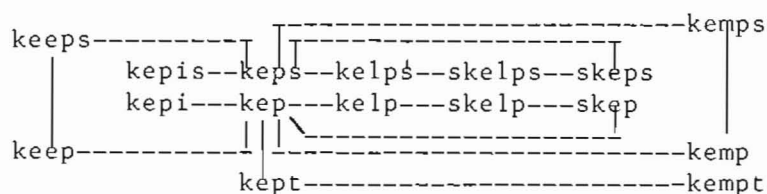
RUCS-**ruc**-ruck-truck-tuck-stuck-suck-**suk**-souk-sou -so-soy-oy-boy-
-soup-sop-scop-

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bo–boo–book–brook–
scoop–coop–coo–cook–crook–rook–rok–rock–crock–cock–clock–lock–flock–
                                                                      FLOC
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There is no proof that the distances shown in the above sequence can not be shortened using 6–letter words. It is certain that other 4–letter words can be joined in via 5–letter words plus 6–letter and maybe even 7–letter words. The combined effect would probably lengthen the span.

All told, of the 8056 words used in this study, 533 did not join the network. The 3–letter words were accounted for earlier. 452 of the others are 4–letter words. Most of them are islands, but without having included the full 5–letter word list in the database, we cannot be certain of this. Of the 3149 5–letter words which were included, 31 did not join the network, even though they each joined at least two 4–letter words. Here is how some of them are joined:

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luff–bluff–buff    dull–dully–duly    dust–dusty–duty    luck–pluck–puck

hypes–hyps–hypos    jiffy–iffy–miffy
hype––hyp––hypo     jiff        miff
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keeps–––––––––––, ,––––––––––––––––––––––kemps
     |           T |T–––––––––––––––––––T        |
     |      kepis––keps––kelps––skelps––skeps    |
     |      kepi–––kep–––kelp–––skelp–––skep     |
     |          |||\–––––––––––––––––––––|       |
keep–––––––––––––'|––––––––––––––––––––––––––––kemp
          kept––––––––––––––––––––––––––––––kempt
```

If words of more than 5 letters were included in the network, the corresponding span would undoubtedly be greater than 26. I may be able to determine it with a modification of my present method, although my database may not be large enough to give a true picture of the English language.

## Some Specific ID Chains

In the following, the longer words did not come from a computer search; I simply joined them to 5–letter or 4–letter words and let the computer do the rest. Many connections between 3–letter words can be made via either a 2–letter or a 4–letter word. I chose the usually more familiar 4–letter one; however, it was often not possible to avoid using obscure 2–letter words. With Carrollian ladders, one can usually avoid obscure words by allowing slightly longer paths; with ID, the longer path usually leads to even more obscure 2–letter words.

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FARMER'S JOB grow–row–ow–owe–we–wet–whet–wheat
             milk–mil–mail–ail–ai–ait–at–gat–goat
             milk–mil–moil–oil–coil–col–cowl–cow
MILLER'S JOB wheat–heat–het–et–ret–re–ore–fore–for–four–flour
             grind–grin–gin–in–ion–on–con–corn
DAIRYMAN'S JOB skim–ski–skid–kid–id–mid–mild–mil–milk
BREWER'S JOB hops–hop–hope–hoe–he–hae–hale–ale
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BUTCHER'S JOB lamb-lam-lamp-amp-samp-sap-soap-sop-scop-cop-cops-chops
COOK'S JOB bake-ake-ae-are-re-red-read-bread
            pork-porky-poky-pokey-poke-oke-oe-obe-be-ben-bean-beans
            roast-oast-oat-at-eat-et-bet-beet-bee-beef
MOTHER'S JOB dinner-diner-dine-die-dire-ire-re-ere-sere-serve-served
            kids-ids-bids-bis-bits-bit-bait-bat-bath-bathe-bathed
PLUMBER'S JOB sink-sin-in-ion-on-own-ow-owe-we-ewe-ewer-sewer
SHRINK'S JOB daft-aft-at-ant-an-ane-sane
LAUNDRY JOB grey-gey-gley-ley-fley-fly-flay-fay-fa-fat-fiat-fit-it-
            hit-whit-white
CAPTAIN'S JOB ship-hip-hi-phi-pi-poi-po-pot-port
PEOPLE'S JOB end-en-hen-he-hue-huge-huger-hunger
            save-ave-have-hae-hale-whale-whales
            save-ave-ae-are-re-ore-fore-fores-forest-forests

FROWN-frow-row-ow-mow-mo-mol-moil-mil-mile-SMILE
ACE-acne-ane-aine-ain-akin-kin-KING
KILL-ill-till-til-ti-tie-te-ate-pate-pater-parter-PARTNER
TWAIN-tain-tan-ta-eta-meta-met-MEET
DRUG-rug-rung-run-rune-rue-re-ore-tore-STORE
CANDY-cany-can-cant-cat-at-tat-teat-TREAT
BIRDS-bird-bid-id-lid-laid-lad-lady-lay-flay-FLY
FOREST-fores-ores-ore-re-ree-tree-TREES
BLUE-bluer-blur-lur-lure-ure-re-are-acre-ace-lace-lac-lack-BLACK
WATER-waster-waste-wast-wat-want-wan-wain-win-WINE
RED-reed-ree-gree-GREEN
BEES-benes-bens-ens-lens-liens-lins-lings-ling-sling-sing-STING
BRAWN-bran-brain-BRAINS
RAIN-rin-in-sin-sine-SHINE
LEAST-east-eat-meat-mat-moat-mot-MOST
BEAST-bast-bas-bans-ban-an-MAN
CROSS-coss-cos-cods-cod-od-rod-ROAD
SLOW-sow-so-sot-st-sit-sift-SWIFT
BITCHY-bitch-itch-pitch-pith-pit-pi-pie-pice-ice-NICE

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                  mow----------------------------ow
                  mo                    hour      owe
MONTH--moth--mot---mort--ort--or--our---your       |
            moat              DAY       you         |
              |                          |          |
    eat--at--mat---ma--may--ay--ayu-----yu          |
    ea                                              |
    ear-----YEAR---yea---ye---wye------------we--wee--WEEK

                          ami--MI
or---ore--RE--are---ar---air--ai--------------------ain
dor                  far          |                  |
DO---ado--ad--fad---FA            |                  |
         lad         fat          |                  |
         LA--lat----at-------ait--it--tit--TI--tin--in
                     sat          |                  |
                   st--sot--SO--son                  |
don----------------------------------on-------------ion
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