# ALPHAMAGIC SQUARES

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In the November 1990 Kickshaws, Dave Morice presented some remarkable word sets due to Michael Sussna. As Ross Eckler was kind enough to point out, the latter are closely related to **alphamagic squares**, a topic I introduced in a two-part article that appeared in 1986 in Abacus, a computer quarterly published by Springer-Verlag [1]. Alas, Abacus was discontinued in 1988 after a brief five-year run. (I like to think this was not due to my contributions!) As a result, Abacus is little known, and back issues hard to come by, a fact perhaps reflected in Morice's unawareness of alphamagic squares. This seems a pity, not only for the fame it denies the author, but also because of the intrinsic interest of the topic and the opportunities it offers for breaking new ground, both to computerists and pencil-pushers alike among logologists. In hopes of stimulating interest among Word Ways readers, I present a brief synopsis of the article cited above. Perhaps some of the unanswered questions it raises will whet appetites.

## Alphamagic Squares

A square is "magic" when the sums of its numbers in every row, column and diagonal are the same. The following square of order 3 (meaning 3-by-3) is both "magic" and "alphamagic" -- the term I use for meta-magic squares whose number-words produce equal row, column and diagonal letter totals also (given at the right).

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 22 | 18 | five | twenty-two | eighteen | 4 | 9 | 8 |
| 28 | 15 | 2 | twenty-eight | fifteen | two | 11 | 7 | 3 |
| 12 | 8 | 25 | twelve | eight | twenty-five | 6 | 5 | 10 |

Thus, in the top row, 5+22+18 = 45, the ordinary magic sum, while **five, twenty-two** and **eighteen** have 4, 9 and 8 letters respectively, yielding an alphamagic sum of 21. The array at the right shows the second magic square resulting from these letter counts or **logs** (short for logorithms) as I call them. Hence log 1 = 3, log 3 = 5, etc. Note that in this case the logs are distinct and minimal (3 being the smallest log in English), as well as consecutive: 3,4,5, 6,7,8,9,10,11. Moreover, this unusual gem (a magic spell known as the **Li** Shu) is the very **first** English alphamagic square.

What does first mean? On the next page, I show the first ten alphamagics of order 3 (for brevity, number-words have been omitted); rotations and reflections of the same square are counted identical. Squares using repeated numbers are judged trivial, while repetitions among logorithms are tolerated. The ten are put in se-

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. 1 | 5 | 22 | 18 | 4 | 9 | 8 | | No. 6 | 4 | 101 | 57 | 4 | 13 | 10 |
| | 28 | 15 | 2 | 11 | 7 | 3 | | | 107 | 54 | 1 | 15 | 9 | 3 |
| | 12 | 8 | 25 | 6 | 5 | 10 | | | 51 | 7 | 104 | 8 | 5 | 14 |
| No. 2 | 8 | 19 | 18 | 5 | 8 | 8 | | No. 7 | 44 | 61 | 57 | 9 | 8 | 10 |
| | 25 | 15 | 5 | 10 | 7 | 4 | | | 67 | 54 | 41 | 10 | 9 | 8 |
| | 12 | 11 | 22 | 6 | 6 | 9 | | | 51 | 47 | 64 | 8 | 10 | 9 |
| No. 3 | 15 | 72 | 48 | 7 | 10 | 10 | | No. 8 | 5 | 102 | 58 | 4 | 13 | 10 |
| | 78 | 45 | 12 | 12 | 9 | 6 | | | 108 | 55 | 2 | 15 | 9 | 3 |
| | 42 | 18 | 75 | 8 | 8 | 11 | | | 52 | 8 | 105 | 8 | 5 | 14 |
| No. 4 | 18 | 69 | 48 | 8 | 9 | 10 | | No. 9 | 45 | 62 | 58 | 9 | 8 | 10 |
| | 75 | 45 | 15 | 11 | 9 | 7 | | | 68 | 55 | 42 | 10 | 9 | 8 |
| | 42 | 21 | 72 | 8 | 9 | 10 | | | 52 | 48 | 65 | 8 | 10 | 9 |
| No. 5 | 21 | 66 | 48 | 9 | 8 | 10 | | No. 10 | 46 | 78 | 101 | 8 | 12 | 13 |
| | 72 | 45 | 18 | 10 | 9 | 8 | | | 130 | 75 | 20 | 16 | 11 | 6 |
| | 42 | 24 | 69 | 8 | 10 | 9 | | | 49 | 72 | 104 | 9 | 10 | 14 |

quence firstly by magic constant, and secondly by the lowest number occurring: 2 in the first, 5 in the second, etc. Applicable to every order, a unique index number can be thus attached to every square. Where the lowest numbers of different squares coincide, ranking will depend on the second lowest, etc. As with ordinary magic squares, standard practice is to orient specimens so that the smallest corner number appears top left, with the smaller of its two immediate orthogonal neighbors in the top row (middle cell for order 3). Where different squares employ identical numbers, as may occur with higher orders, this latter convention determines rank. As we see, the **Li Shu** is the first on the list, and the only square to show serial logs. Do others exist?

Consider what happens if every entry in the **Li Shu** is increased by 100. Thus **five** becomes **one hundred five**, etc. The addition of a constant to both numbers (100) and letters (10 = log 100) means the resulting matrix is again alphamagic. I call this the second **harmonic** of the **Li Shu**; it is square No. 17. Using **two hundred** instead would produce the third harmonic (No. 26), and so on, all of them showing serial logs. Aside from harmonics, however, squares No. 91 and 120, given below, are the only two further cases among the 217 English alphamagics constructible from numbers under 500.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 215 | 372 | 298 | 17 | 22 | 21 | | 249 | 320 | 328 | 19 | 18 | 23 |
| 378 | 295 | 212 | 24 | 20 | 16 | | 378 | 299 | 220 | 24 | 20 | 16 |
| 292 | 218 | 375 | 19 | 18 | 23 | | 270 | 278 | 349 | 17 | 22 | 21 |

Look next at square No. 7 above. The distribution of 1's, 4's and 7's in the units position of every entry has a curious consequence. Due to the chance that log 1 = log 2, log 4 = log 5, and log 7 = log 8, adding one to every number in the matrix results in a second alphamagic square: No. 9. Squares No. 6 and No. 8 are similarly related. There are sixteen of these pairs -- some adjacent, some more widely separated -- among the first one hundred squares.

Square No. 7 has another interesting property. Its logorithm square, written out in words, is given below. Seen thus, a natural question arises: could this square be alphamagic (albeit trivial) too? The answer, of course, is yes, its logs being as shown at the right. At this point it is impossible not to wonder whether this new square is in turn alphamagic itself, but alas, repetition of the process yields only a semi-magic derivative.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 10 | nine | eight | ten | 4 | 5 | 3 |
| 10 | 9 | 8 | ten | nine | eight | 3 | 4 | 5 |
| 8 | 10 | 9 | eight | ten | nine | 5 | 3 | 4 |

## Exotic Squares

An interesting prospect opens up here that ambitious readers may like to follow up. Ideally, of course, we seek a square giving rise to an unbroken chain of alphamagic derivatives, culminating, as any such chain eventually must do, in a closed loop of repeated squares. The shortest possible loop would be of length 1: a self-reproducing square showing nine 4's, since log 4 = 4. My research shows that the first few hundred English squares give rise only to short chains. However, opportunities for making interesting finds broaden enormously when we include squares in different languages. What is the longest chain using **multilingual** links, for instance? This brings us to foreign language squares in general.

Here is French order 3 magic square No. 14 (magic constants 336; 27), the first in French to show serial logs. The figures in brackets are the number-word values followed by their logs.

| | | |
|---|---|---|
| quinze (15;6) | deux cent six (206;11) | cent quinze (115;10) |
| deux cent douze (212;13) | cent douze (112;9) | douze (12;5) |
| cent neuf (109;8) | dix huit (18;7) | deux cent neuf (209;12) |

Next comes Latin square No. 4 (magic constants 411; 60), using odd numbers only, and Italian square No. 3 (magic constants 381; 45), the lowest serial log squares in these languages.

| | | |
|---|---|---|
| septem et nonaginta (97;17) | centum septuaginta quinque (175;24) | centum triginta novem (139;19) |
| centum septuaginta novem (179;22) | centum triginta septem (137;20) | quinque et nonaginta (95;18) |
| centum triginta quinque (135;21) | novem et nonaginta (99;16) | centum septuaginta septem (177;23) |

| | | |
|---|---|---|
| ottantasette (87;12) | cento sessantacinque (129;14) | cento ventinove (129;14) |
| cento sessantanove (169;17) | cento ventisette (127;15) | ottantacinque (85;13) |
| cento venticinque (125;16) | ottantanove (89;11) | cento sessantasette (167;18) |

German square No. 72 (magic constants 165; 42) is one of many to reveal uniform logs; more significantly, it is a **translation** of English square No. 9!

| | | |
|---|---|---|
| fünfundvierzig (45;14) | zweiundsechzig (62;14) | achtundfünfzig (58;14) |
| achtundsechzig (68;14) | fünfundfünfzig (55;14) | zweiundvierzig (42;14) |
| zweiundfünfzig (52;14) | achtundvierzig (48;14) | fünfundsechzig (65;14) |

This opened a new branch to explore in the logological labyrinth. I discovered, for example, that Welsh square No. 12 (magic constants 216; 33) translates into Norwegian square No. 12 (using even numbers only), the first showing consecutive logs. Alas, space prohibits a fuller report on exotic alphamagics and their interrelations; interested readers are referred to the original article [1].

| | | |
|---|---|---|
| chwech deg dau (62;12) | wyth deg (80;7) | saith deg pedwar (74;14) |
| wyth deg pedwar (84;13) | saith deg dau (72;11) | chwech deg (60;9) |
| saith deg (70;8) | chwech deg pedwar (64;15) | wyth deg dau (82;10) |

| | | |
|---|---|---|
| sekstito (62;8) | atti (80;4) | syttifire (74;9) |
| attifire (84;8) | syttito (72;7) | seksti (60;6) |
| sytti (70;5) | sekstifire (64;10) | attito (82;6) |

## Higher Orders

So far I have said nothing about how the squares presented above have been discovered. No one will be surprised that a computer program was used, full details of which are available in [1]. Suffice it to say that the program design owes much to an algebraic formula describing the general form of every 3-by-3 magic square. However, readers who may be regretting that most of the worthwhile nuggets have already been grabbed from the alphamagical goldfield are in for a pleasant surprise. This is because order 3 squares are a special, unusually tractable, case. In coming to higher orders, it turns out that serious obstacles beset the programmer's path. Leaving aside number-crunching on a juggernaut scale, I at least have been unable to come up with a workable program able to sift for solutions as in the case of order 3. Of course, others may yet succeed where the author has failed.

Facing defeat in attempting to comb systematically for larger squares, the problem of how to produce even a single order 4 square thus waxed in importance. That some progress was made can be seen from the square below, constructed by pencil and paper methods without using a computer. Note that the magic constants are 58 and 29, and that 58 = 29x2. Again space denies fuller explanation,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 18 | 12 | 23 | 5 | | 8 | 6 | 11 | 4 |
| 3 | 25 | 19 | 11 | | 5 | 10 | 8 | 6 |
| 16 | 13 | 1 | 28 | | 7 | 8 | 3 | 11 |
| 21 | 8 | 15 | 14 | | 9 | 5 | 7 | 8 |

although I can say that construction started with a **Latin** square which was then modified step by step, alphamagicality being preserved at each stage until all entries were rendered distinct. This square is a cunningly-disguised **Graeco-Latin** square, one consequence of this heritage being that cyclic permutations and other transposals of the numbers in the matrix can produce no fewer than 144 different alphamagic squares. Graeco-Latins splay a fundamental part in alphamagic squares, including those of order 3, but again I shall have to refer readers to the original article for full details. Notice that as a result of the construction method, the index numbers of all those 144 squares remain unknown.

What ought to be clear by now is that completion of higher-order squares is anything but a process of straightforward calculation, whether by machine or hand. Thus far no recipes exist; alphamagic square construction demands creativity. The reader is thus contemplating almost virgin territory. For the intrepid, rich pickings are doubtless to be had. Perhaps I can emphasize this by presenting the only other non-trivial order 4 square produced to date, with magic constants 77 and 32. The use of zero is felicitous.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 31 | 23 | 8 | 15 | | 9 | 11 | 5 | 7 |
| 17 | 5 | 21 | 34 | | 9 | 4 | 9 | 10 |
| 26 | 38 | 13 | 0 | | 9 | 11 | 8 | 4 |
| 3 | 11 | 35 | 28 | | 5 | 6 | 10 | 11 |

And lastly, the fruit of pensive nights, the square below shows an inner alphamagic square of order 3.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 59 | 89 | 17 | 44 | 61 | | 9 | 10 | 9 | 9 | 8 |
| 67 | 4 | 101 | 57 | 41 | | 10 | 4 | 13 | 10 | 8 |
| 15 | 107 | 54 | 1 | 93 | | 7 | 15 | 9 | 3 | 11 |
| 82 | 51 | 7 | 104 | 26 | | 9 | 8 | 5 | 14 | 9 |
| 47 | 19 | 91 | 64 | 49 | | 10 | 8 | 9 | 9 | 9 |

In conclusion, therefore, a number of as-yet-unfulfilled alphamagic challenges are worth listing. Can any reader produce

1) Any interesting new order 3 finds?
2) Any new alphamagics of order 4 or higher?
3) Higher-order squares in foreign languages?
4) Squares that translate into each other?
5) The index numbers of the order 4 squares above?
6) An order 4 square whose log square is also alphamagic?
7) An alphamagic cube?

Interested readers can receive a copy of [1], the full version of this synopsis, by writing to the author at Buurmansweg 30, 6525 RW Nijmegen, The Netherlands (telephone (0)80-562601).

REFERENCE

[1] Sallows, Lee, Alphamagic Squares, Part I, Abacus, Vol. 4, No. 1, Fall 1986; Alphamagic Squares, Part II, Abacus, Vol. 4, No. 2, Winter 1987.

## *A HISTORY OF RIDDLES*

*Beginning with Will Shortz's 1973-75 articles on British and American word puzzles of the eighteenth and early nineteenth centuries,* **Word Ways** *has occasionally (dare one say periodically?) examined the roots of wordplay: The Greek Anthology of the 10th century containing various classical riddles (1985), The Exeter Book of Anglo-Saxon English riddles (1977), Tabouret's* **Les Bigarrures et Touches**, *the first book entirely devoted to recreational linguistics (1979, 1980). All of this and much more is described by Mark Bryant in his landmark* **Dictionary of Riddles** *(Routledge, 1990; $25), a 366-page survey of the riddle containing nearly 1500 specimens spanning more than two millennia. His definition of "riddle" is broad, encompassing not only a description of a common object in a deliberately misleading or cryptic manner ("What we caught we threw away; what we could not catch we kept"), but also many of the puzzle types found in The Enigma.*

*Bryant asserts that the riddle, viewed as a pastime seriously pursued by educated adults, reached its zenith in the time of Schiller (1759-1805) and has since degenerated (except in the miniworld of the National Puzzlers' League) to the role of a children's amusement. Why the decline? Bryant doesn't know, but observes that "interest in riddles seems to coincide with seasons of intellectual awakening...the riddle has always appeared hand in hand with a resurgence in cultural activity, almost as a touchstone of cultural finesse." However, association is not causation, and the question "What conditions encourage riddling?" is still an open one. Perhaps that quality of mind that embraced riddling for so many centuries has now abandoned it in favor of the crossword puzzle, Rubik's cube, or even video games such as Dungeons and Dragons. If so, it's a pity, for in their heyday riddles evolved into crafted literary works of art: Bryant lovingly recreates their now-vanished glories. For all those interested in wordplay history, this book is a must.*

*Oh, by the way, the answer to the riddle above is fleas.*