SPANNING THE GLOBE WITH THE SCRABBLE® TILES

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Here is a puzzle that's simple to state but not so easy to solve. Take a set of English Scrabble tiles with its standard complement of 98 letters and two blanks (that can be used to stand for any letter). Use all 100 tiles to make a list of names of countries in the world, all different. For the acceptable list of country names use the current list of 192 member states of the United Nations. (Note that, as indicated by the list given on the UN website, the official name of the country usually called "Ivory Coast" in English is "Côte d'Ivoire". We will use the latter designation in what follows.)

Does this puzzle have a solution? We will present some results of investigating this question starting on the next page, so if you wish to give it a go yourself wait until you've worked on the puzzle to turn the page. If you can't use all the tiles, how close can you come? How small can the sum of the points on the unused tiles be?

Attack Strategy

This is a hard puzzle to solve by hand and also quite challenging with computer aid because of the vastness of the space of configurations that much be searched. Guessing that the average length of the countries in our final list might be 7 letters, this means there will be roughly 14 countries in the list. So a straightforward search would need to examine $C(192,14)$ combinations to see if each can be formed with the Scrabble tiles, a number which is about half a sextillion. The number of combinations can be reduced significantly by the observation that when we have chosen the first n countries in our list, the countries available for filling in the remainder of the list are just those which can be formed from the currently unused tiles (including any blanks that remain as well). As we work through the list, the set of usable countries gets smaller and smaller, reducing the size of the search tree.

Unfortunately, even taking into account this observation the search space is still too large to examine exhausitvely in a reasonable amount of computer time. However, there is another technique we can employ that will bring it into the range of feasibility. The idea is to select countries that use up the rarer letters of the alphabet first. If we're going to be successful in using all the Scrabble tiles, the rare letters (like J, K, Q, X, and Z, which appear just once in the Scrabble set) have to be chosen *some time*, so we might as well choose them at the beginning. Since these letters are rare there will only be a few country names containing them, which reduces the number of choices greatly in the early stages of the search. If we choose an X word first, for example, then there are only two choices for the first country (Luxembourg and Mexico) instead of 192.

We decided to choose the first six countries this way, corresponding to the rare letters J, K, Q, X, Z and F. This reduces the number of choices for the first six levels of the search tree from (192, 191, 190, 189, 188, 187) to (3, 8, 2, 2, 9, 4), a reduction by a factor of about 12 trillion. The rest of the countries can be chosen by exhaustive search augmented by the observation in our third paragraph. Such a program, executed on an ordinary 2.8 GHz PC, was able to complete its search in about three and a half hours.

Some Solutions

Not having been successful in solving this puzzle by hand (or even coming close) we really didn't know what to expect, and so were pleasantly surprised to find no fewer than 569 different solutions. These are not necessarily all the solutions, as took a few shortcuts here and there to reduce the execution time of the program, but we expect the number of solutions missed to be quite small.

The fewest number of countries in any of our lists is 13, and there are just two lists with this number:

Cote d'Ivoire Egypt Ethiopia Fiji Lebanon Luxembourg Maldives Montenegro Qatar SouthAfrica Swaziland Sweden Turkey
Benin Cote d'Ivoire Egypt ElSalvador Ethiopia Fiji Luxembourg Montenegro Qatar SouthAfrica Swaziland Sweden Turkey

(Underlined letters designate the blank tiles.) We found seven lists that achieve the maximum of 18 countries. In just one of these, all the country names have seven letters or less:
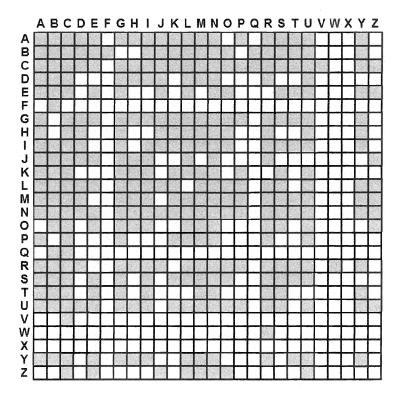
Belize Egypt Estonia Fiji France Ghana India Iraq Lesotho Mexico Moldova Peru Rwanda Serbia Sweden Togo Turkey Tuvalu

There are 42 countries which do not appear in any list, and 10 that only occur in one: Bahamas, Bulgaria, Burkina Faso, Canada, Colombia, Macedonia, Morocco, Pakistan, Sierra Leone, and Turkmenistan. Here, for example, are the unique lists for Burkina Faso, Canada, and Sierra Leone:

Bolivia Burkina Faso Egypt Finland Greece Haiti Iraq Jordan Lesotho Mexico Peru Sweden Switzerland Togo Tuvalu Yemen
Belize Canada Cote d'Ivoire Egypt Fiji France Honduras Iraq Kuwait Lesotho Luxembourg Nepal Sweden Syria Togo Vietnam
CapeVerde Egypt Fiji Finland Iraq Kuwait Lesotho Mexico Moldova Niger SierraLeone SouthSudan Tonga Turkey Zimbabwe

There are 351 possibilities for the assignment of letters to blanks, and we were able to find solutions with 173 (49%) of these combinations. The combinations we found are shown as shaded squares in the diagram to the right. As indicated by the empty column/row, the only letter that never appears on a blank in our solutions is X. A blank standing for X is not impossible *a priori*, because if one could manage to use the two X names (Mexico and Luxembourg) in the same list then one of the X's would have to be a blank, but we did not find such a solution.

Of the 26 double-letter possibilities for the blanks (AA, BB, ..., ZZ), exactly half of them occur in our lists. Perhaps most remarkable are the two lists that use KK, one of which is shown below.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A																										
B																										
C																										
D																										
E																										
F																										
G																										
H																										
I																										
J																										
K																										
L																										
M																										
N																										
O																										
P																										
Q																										
R																										
S																										
T																										
U																										
V																										
W																										
X																										
Y																										
Z																										

Belize CapeVerde Fiji France Haiti Lesotho Luxembourg Norway Qatar Slovakia Spain Sweden Togo Turkey UnitedKingdom

Also interesting is this solution, with the two blanks standing for the rare letters J and Z:

Belize Cote d'Ivoire Egypt Fiji Iraq Jordan Kenya Lesotho Luxembourg Peru SouthAfrica Swaziland Sweden Tonga Vietnam

The country contained in the most number of solutions is Fiji, which appears in a whopping 562 of 569 lists. This is not unexpected, since with just four tiles the name "Fiji" captures two of the rare letters (F and J). The next most frequently used countries are Sweden (540 lists), Egypt (651), Lesotho (470), Togo (388), Iraq (365), and Côte d'Ivoire (351). If we use the name "Ivory Coast" instead of "Côte d'Ivoire" the number of solutions containing that country is drastically reduced, to about 30.

Close observation of the set of solutions leads to the realization that some of them are related to each other in an interesting way. Consider this list, for example, that contains 15 countries and has the blanks assigned to C and U:

Belize Congo Cote d'Ivoire Egypt Ethiopia Fiji France Luxembourg Rwanda Slovenia SouthSudan Sweden Turkey **Iraq Malta**

We have emphasized the last two countries in the list because they happen to be part of one of the four *two-element subset anagrams* which exist within the set of country names:

$$IRAQ + MALTA = MALI + QATAR$$
$$GABON + ITALY = LIBYA + TONGA$$
$$BELARUS + INDIA = LIBERIA + SUDAN$$
$$ISRAEL + UGANDA = ALGERIA + SUDAN$$

So this list can be transformed into another valid list by changing the subset anagram:

Belize Congo Cote d'Ivoire Egypt Ethiopia Fiji France Luxembourg Rwanda Slovenia SouthSudan Sweden Turkey **Mali Qatar**

and, indeed, this second list also was also found by our search. In fact, all four of the two-element subset anagrams appear somewhere in our set of solutions. Many pairs of solutions are related in this way, sometimes with subset anagrams having more than two elements. The most extreme case is this list of 15 countries containing a 13-element subset anagram:

Fiji Egypt **Belize Burundi CapeVerde Estonia Iraq Kuwait Mexico Moldova Netherlands Norway Senegal SouthAfrica Togo**
Fiji Egypt **Cote d'Ivoire France Gabon Haiti Lesotho Luxembourg Macedonia Peru Qatar Slovenia Swaziland Sweden Turkey**

Some Related Puzzles

The pair of solutions just above, which between them contain 28 different countries, suggests an intriguing challenge. Take two sets of Scrabble tiles and make a list of countries with each. Is it possible for the countries in both lists combined to be all different? We tentatively conjecture that the answer is "no", but it is possible to get tantalizingly close, as several pairs from our 569 solutions have just one country in common. The shared country is always either Fiji or Sweden; here is one pair for each case:

Belarus Belize CapeVerde CoteDlvoire Egypt FIJI Finland Germany Iraq Kuwait Lesotho Mexico Norway SouthSudan Tonga
Bolivia Chad Congo Estonia Ethiopia FIJI France Luxembourg Maldives Peru Qatar Sweden Switzerland Togo Turkey Yemen

Djibouti Egypt Finland France Haiti Iraq Kuwait Lesotho Mexico Moldova Niger Peru Serbia SWEDEN Syria Togo Venezuela
Congo CoteDlvoire Ethiopia Fiji Gabon Luxembourg Maldives Nepal Qatar SouthAfrica SWEDEN Switzerland Turkey Yemen

Each of these list pairs contains a total of 30 different countries, the current record.

Another related puzzle comes from noting that the largest number of countries in any of our lists is 18. Is it possible to make more than 18 countries if we are not required to use all the tiles? The answer is yes - here is a list made with 97 tiles containing 19 countries:

Belize Chad Egypt Fiji India Iraq Laos Lethoso Mexico Niger Oman Peru Rwanda Serbia Sweden Togo Tonga Turkey Tuvalu

The unused tiles are E,F,V, totalling 9 points, which we conjecture is the minimum possible.

Gerunding
by Ron Singer

Bringing my knitting to meeting.
The meeting's on reading.
Knitting for meeting,
meeting for reading.
Reading or knitting?
Knitting for reading.
Needing to meet,
needing to knit,
meaning to read
about needing to knit.