5X5X5 WORD CUBES BY COMPUTER

DONALD E. KNUTH
Stanford, California

Recently, I used a computer to look for 5x5x5 word cubes: symmetric patterns that spell five-letter words in rows, columns and whatchamacallits (is there a name for vertical 1x1x5s?). As far as I know, the only previous example was constructed by Peter Graham and published in Omni in July 1987. But his solution

L O V E D	O P E R A	V E L A R	E R A S E	D A R E S
O P E R A	P U R E R	E R O D E	R E D A N	A R E N A
V E L A R	E R O D E	L O G O S	A D O R E	R E S E T
E R A S E	R E D A N	A D O R E	S A R U M	E N E M Y
D A R E S	A R E N A	R E S E T	E N E M Y	S A T Y R

uses REDAN and SARUM, which are not in my vocabulary. Also VELAR is kind of hazy for me. I know it's a word, but I don't think I've seen it in novels or "real" English.

Lots of obscure five-letter combinations can be found in unabridged dictionaries, but word puzzles are much more fun when they stick to words that I wouldn't feel like challenging if my opponent played them in Scrabble. So I've been collecting a list of all five-letter words that I personally know and love, during odd moments since the early 1970s. Last week I decided to see if any 5x5x5 cubes were possible using only words from my collection (5757 in all) ... and I found to my surprise that there are many, many solutions (exactly 83576)!

Here are the five best, as far as I can see:

A S T E R	S C A L E	T A C O S	E L O P E	R E S E T
S C A L E	C O D E X	A D A P T	L E P E R	E X T R A
T A C O S	A D A P T	C A P E R	O P E R A	S T R A P
E L O P E	L E P E R	O P E R A	P E R K S	E R A S E
R E S E T	E X T R A	S T R A P	E R A S E	T A P E R

T Y P E S	Y E A S T	P A S T A	E S T E R	S T A R T
Y E A S T	E A R T H	A R M O R	S T O L E	T H R E E
P A S T A	A R M O R	S M O K E	T O K E N	A R E N A
E S T E R	S T O L E	T O K E N	E L E C T	R E N T S
S T A R T	T H R E E	A R E N A	R E N T S	T E A S E

A B L E R	B L A D E	L A P I S	E D I C T	R E S T S
B L A D E	L O G I N	A G E N T	D I N A R	E N T R Y
L A P I S	A G E N T	P E S T O	I N T R O	S T O O L
E D I C T	D I N A R	I N T R O	C A R G O	T R O O P
R E S T S	E N T R Y	S T O O L	T R O O P	S Y L P H

```
O T H E R   T H E M E   H E R O S   E M O T E   R E S E T
T H E M E   H E R O N   E R U P T   M O P E R   E N T R Y
H E R O S   E R U P T   R U L E R   O P E R A   S T R A P
E M O T E   M O P E R   O P E R A   T E R M S   E R A S E
R E S E T   E N T R Y   S T R A P   E R A S E   T Y P E S

A F T E R   F R A M E   T A L C S   E M O T E   R E S E T
F R A M E   R I G O R   A G O R A   M O R N S   E R A S E
T A L C S   A G O R A   L O Y A L   C R A S S   S A L S A
E M C E E   M O R N S   C R A S S   E N S U E   E S S E S
R E S E T   E R A S E   S A L S A   E S S E S   T E A S E
```

Each of these has one somewhat unusual word: CODEX, ESTER, LOGIN, MOPER, TALCS. But they all pass the Scrabble test, except that LOGIN might not yet be in standard dictionaries ("What is your login name?").

Another solution uses no unusual words whatever, but it has a flaw in that it uses ERASE twice (or more times, depending on how you count):

```
R A C E R   A D O R E   C O M A S   E R A S E   R E S E T
A D O R E   D I V A N   O V E R T   R A R E R   E N T R Y
C O M A S   O V E R T   M E T E R   A R E N A   S T R A P
E R A S E   R A R E R   A R E N A   S E N D S   E R A S E
R E S E T   E N T R Y   S T R A P   E R A S E   T Y P E S
```

Notice that the final square here is the same as in the fourth solution above.

The vast majority of the 83576 solutions contain too many rare words to be really interesting.

Why are there so many solutions? Because each solution can usually be tweaked into another by changing only a few letters. For example, you can easily find 20 different solutions that fill in the *s of

```
* A * E R   A D O R E   * O * A S   E R A S E   R E S E T
A D O R E   D I V A N   O V E R T   R A R E R   E N T R Y
* O * A S   O V E R T   * E * E R   A R E N A   S T R A P
E R A S E   R A R E R   A R E N A   S E N D S   E R A S E
R E S E T   E N T R Y   S T R A P   E R A S E   T Y P E *
```

and 57 ways to complete the pattern

```
* S * E R   S C A L E   * A * O S   E L O P E   R E S E T
S C A L E   C O D E X   A D * P T   L E P E R   E X T R A
* A * O S   A D * P T   L * * E *   O P E R A   S T * A *
E L O P E   L E P E R   O P E R A   P E R K S   E R A S E
R E S E T   E X T R A   S T * A *   E R A S E   T A * E *
```

Here is a list of common words to fill in the asterisked patterns: *O*AS (sodas, sofas, novas, comas, yogas, togas, iotas, colas, codas, bolas, cocas, somas, soyas, molas); *ASER (laser, baser, maser); *ANER (saner, caner); *ACER (racer, pacer, facer, macer, lacer); *AYER (layer, gayer, payer, sayer, hayer); *ATER (water,

later, cater, eater, mater, hater, pater, rater, tater, dater); *ABER (saber); *AMER (tamer, lamer, namer, gamer); *E*ER (never, refer, meter, fewer, fever, lever, newer, sewer, deter, sever, leper, defer, peter, seder, hewer, weber, ceder, hexer, keyer); TYPE* (types, typed). I have ranked the words in my corpus by frequency of use in various texts; the common words listed above have ranks of 3500 or less. If the entire list of 5757 words is used, I get not 20 but 110 different solutions. The point here is not to obtain an exact count, but rather to illustrate the multiplicative phenomenon that accounts for the large number of word cube solutions.

If my stockpile of five-letter words is restricted to the most common 3000, there are no word cubes possible. Expanding the list to 3500 results in 83 solutions, including the one in the middle of the preceding page.

All but 23 of the 5757 words participate in at least one word cube, the exceptions being JELLY, JOLLY, JUICY, SQUAD, QUILL, SKIFF, GODLY, DIZZY, QUIRK, GUMMY, VYING, GAUZY, WHIZZ, WOOZY, JAZZY, JIMMY, JOWLY, BOOZY, DJINN, SPAZZ, BUZZY, BOOKY and LAWZY (in decreasing order of frequency).

A final note on word squares. The three word squares below can be formed if one uses the commonest 372 words (AGREE having rank 372):

```
G R A S S     G L A S S     C L A S S
R I G H T     L I G H T     L I G H T
A G R E E     A G R E E     A G R E E
S H E E P     S H E E P     S H E E P
S T E P S     S T E P S     S T E P S
```

No further solutions appear until the 711th word, ALARM, which yields the square

```
P A R T S
A L A R M
R A D I O
T R I C K
S M O K E
```