WORDNIM AND GRUNDYWORD

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An interesting two-person game begins by choosing any set of words - say, for example, O, HIP, LLAMA and PIASTER. Players alternately remove one or more letters from exactly one of the four words (all the letters in a word can be removed). Both the deleted letters and those remaining in the player's chosen word must be transposable into words themselves. The game ends when a player cannot make a legal move; the other player is the winner.

Suppose we have accepted your gracious offer to play first with the four words above. We chose to remove the letters TRIPE from PIASTER, leaving the set O, HIP, LLAMA and AS. There is now only one move you can make that ensures you a win. Can you find it?

We are playing Wordnim, a word game based on the classic combinatoric game of Nim. Nim is played with several piles of counters (such as coins or matchsticks). A player on move must take one or more counters from exactly one pile. The player who takes the last counter wins. In his book Your Move (McGraw-Hill, 1971; Dover 1991), David L. Silverman describes a winning strategy, which we illustrate by example. Suppose we have piles of size 1, 3, 5 and 7. Any number can always be written as sums of powers of two; in our example, $1 = 1$, $3 = 1 + 2$, $5 = 1 + 4$, and $7 = 1 + 2 + 4$. For any two numbers x and y, define the nim-sum, $x \oplus y$, to be the sum of their powers of two, with the proviso that $2^n \oplus 2^n = 0$ for any n. Thus, $5 \oplus 7 = 1 \oplus 4 \oplus 4 \oplus 2 \oplus 1 = (4 \oplus 4) + (1 \oplus 1) \oplus 2 = 0 \oplus 0 \oplus 2 = 2$. So, $5 \oplus 7 = 2$. Similarly, $1 \oplus 7 = 6$, etc. A table of nim-sums is given below.

$\oplus$	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	0	3	2	5	4	7	6
2	2	3	0	1	6	7	4	5
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	5	4	7	6	1	0	3	2
6	6	7	4	5	2	3	0	1
7	7	6	5	4	3	2	1	0

Note that in our example $1 \oplus 3 \oplus 5 \oplus 7 = 0$. A player on move, facing a nim-sum of zero, must play so that the new nim-sum is not zero. His opponent can always play so that the nim-sum is changed back to zero. Silverman gives a nice discussion of the reasons for this in his book. Sooner or later, the first player is faced with the ultimate zero, i.e., no counters at all, and must lose. In our example, suppose the first player removes five counters from the seven-pile. The nim-sum is changed to $1 \oplus 3 \oplus 5 \oplus 2 = 1 \oplus (1 \oplus 2) \oplus (1 \oplus 4) \oplus 2 = 5$. Therefore, to win, the second player takes all of the five-pile, changing the nim-sum back to zero.

The solution in our Wordnim example game is similar; you must take the word LLAMA to maintain your winning position.

Of course, there is more to Wordnim than the simple matematical nim-sum strategy; one must be able to actually form the words one wants to leave and take. This will depend somewhat on the abilities of the players and also on the pool of allowed words. We like to use for our word-source the new (third) edition of the American Heritage Dictionary since it lists abbreviations, symbols, names, etc., alphabetically in boldface type. We also permit any inferred forms to be on our word list. Naturally, one-letter words are acceptable.

The words O, HIP, LLAMA and PIASTER seem to form a particularly fecund list for forming word subsets. Even 'LL (shall, will) is listed in American Heritage! According to "Is A Picture Worth 1000 Words?" in the November 1980 **Word Ways**, PIASTER generates more than 265 words. Even better, "Analyzing Wiretaps" in the May 1984 **Word Ways** reveals that the different subsets of WIRETAPS all form words. In "Neustria" in the February 1982 **Word Ways**, Darryl Francis found that 247 different subsets of NEUSTRIA formed words (Jeff Grant, the WIRETAPS author, was able to find three Francis overlooked). WIRETAPS and NEUSTRIA are the sorts of words ideal for Wordnim; we urge readers to come up with more of them.

There are at least two significant variants of Wordnim. One can allow any set of letters to be removed (whether transposable to a word or not) as long as a word remains. In our game, change HIP to PHI and PIASTER to PIRATES to form the list O, PHI. LLAMA and PIRATES. Then, successive beheadments by one or more letters from any word always leave a word (ATE'S is a possessive, and the others are inferred plurals).

Another variant is what mathematicians call the misere form of the game. Instead of the winner being the player to take the last word, that player now loses. Unexpectedly, the strategy for misere Wordnim remains almost exactly the same as for the standard version. The only change required is toward the end of the game when the winning player must alertly modify his takes in a rather obvious way. A few practice games will make this clear.

We now turn to Grundyword. In the mid-fifties, P. M. Grundy invented the game that bears his name. Starting with a stack of n checkers, two players alternately break a stack into two unequal stacks. A player loses when he has no legal move. If equal stacks were allowed, the game would trivially depend only on the parity of n; the first player would always win if n is even, and always lose if n is odd. With the unequal stack requirement, Grundy's game is complex and quite interesting.

For Grundyword, we start with a word, MOUSETRAP for example, and require the players in turn to split any word into two words of unequal length. From MOUSETRAP, the first player has the option of breaking that word into the splits 8-1, 7-2, 6-3 or 5-4, provided that he can think of American Heritage words accomplishing these.

Suppose we play Grundyword with a MOUSETRAP start. Suppose that we go first, and, unable to think of any other break, split MOUSETRAP into the pedestrian MOUSE-TRAP. It is your turn, and your options are to split MOUSE into a 4-1 or 3-2, or TRAP into a 3-1. We made a serious error with our 5-4 split which you can now exploit with the proper move. Can you find it?

It turns out that, as in Wordnim, the winning strategy is to force a nim-sum of zero on your opponent. However, the nim-values in Grundyword are not simply the word lengths as they were in Wordnim. Richard K. Guy, in his book Fair Play (COMAP, 1989), has computed nim values in the classic Grundy's game for checker stacks up to size 50. In the table below, we adapt Guy's values for Grundyword words up to size 24. (As a practical matter, it is only necessary to memorize this table up to n = 17.) Compute nim-sums as before, but for a word of length n, use the nim value given below the word-length.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	0	1	0	2	1	0	2	1	0	2	1	3	2	1	3	2	4	3	0	4	3	0	4

Note that the nine-letter MOUSETRAP has a nim value of 1. It is not zero, so the first player should be able to break the word so that the nim-sum of the splits is zero. The options are 8-1 (UPSTREAM-O) with nim-sum $2 \oplus 0 = 2$, 7-2 (MAESTRO-UP) with nim-sum $0 \oplus 0 = 0$, 6-3 (MATURE-SOP) with nim-sum $1 \oplus 1 = 0$, and 5-4 (STEAM-POUR) with nim-sum $2 \oplus 0 = 2$. We should have broken into a 7-2 or a 6-3 to force a zero on you. Your three options from our 5-4 are 1-4-4 (nim-sum 0), 2-3-4 (nim-sum 1) and 5-1-3 (nim-sum 3). Therefore, when we left you MOUSE-TRAP you must break MOUSE into a 4-1 to gain the edge (SOME-U) would work. After that break, you will in the game.

Grundyword places a strong premium on one's ability to recognize transposals. It is probably best to experiment with likely starting words in advance of playing in expert games. When we play against each other, we each prepare a set of words and allow the other the choice of starting word. To be fair, we play two games with that word, alternating first move. For example, one of us had prepared the ten-letter word ASTONISHED. The options for the first player are given in the table on the next page.

Break	Nim-Sum	Possible Words	Breaks To Force Zero
9-1	1	DISHONEST-A	7-2-1 or 6-3-1
8-2	2	HEDONIST-AS, SEDATION-SH	7-1-2
7-3	1	HANDSET-ISO, INSTEAD-OHS, DISSENT-HAO	7-1-2
6-4	1	DONATE-HISS, DANISH-TOES, SADIST-HONE, THESIS-DONA, ADONIS-SETH, HONEST-SAID, ANODES-HITS, DESIST-NOAH, ONSIDE-HAST, SHINED-TAOS	4-2-4

Since ten-letter words have a nim value of zero, the preparer of ASTONISHED expects to win this Grundyword game when it is his turn to move second. The question is, can he win it when he must move first? In the actual game, the preparer broke into HEDONIST-AS. The onus is now on the other player to break HEDONIST into a 7-1 split. There seems to be only one split possible using American Heritage words, HISTONE-D, and the other player did not find it.

Grundyword is ideally suited for playing by mail. To initiate postal Grundyword, send your opponent a postcard with three or four possible starting words on it. He chooses one of your words and returns his first break of that word to you. You now respond to his break and also send him a different first break of the word he chose from your list. Of course, many games could be in play on the same postcard, and a time limit should be imposed.

We close with a small quiz. Suppose you are on move in a Grundyword game facing the words O, HIP, LLAMA and PIASTER. What is your best play? See Answers and Solutions.

QUERY

*Deborah Shine, 444 Central Park West, Suite 8B, New York NY 10025 is compiling a book entitled Rhymes for a Reason: The Kid's Book Of Useful Verse, and is looking for mnemonic rhymes or sentences that could be included in such a book. Although this topic is not often covered in **Word Ways**, perhaps readers will be able to supply relevant material.*