NESTED WORD SQUARES

LEONARD GORDON
Tucson, Arizona
A. ROSS ECKLER
Morristown, New Jersey

In the May 1983 **Word Ways**, Helen Motamen describes a word puzzle contest she won in which the object was to construct a five-by-five letter array containing a maximum number of three-letter, four-letter and five-letter words across and down. By placing further requirements on this contest, one can convert it into an interesting logological problem: the construction of a five-by-five word square in which are nested four four-by-four and nine three-by-three word squares. In fact, there are a whole host of problems to consider. The five-by-five word square may be either single (the five different horizontal words are duplicated vertically) or double (there are five different horizontal words, and five different vertical words, ten words in all). If the five-by-five square is single, the embedded squares on the diagonal are also, but the nested squares located above or below the diagonal may be double or defective (contain two identical words in a row). If the five-by-five square is double, there are many possibilities for the thirteen nested squares: each one may be single, double, defective (contains two identical words in a row, or in a column) or hybrid (one or more, but not all, horizontal words match vertical words). The ideal solution - naturally, the hardest one to achieve - is to have a double five-by-five word square containing thirteen nested double squares, each one different.

D A M A N
A N A N A
M A L A R
A N A N A
N A R A S

The prize-winning Motamen array, given at the right, does not qualify as a five-by-five word square because it contains the repeated word ANANA. Her hand search of Webster's Third Unabridged (the contest dictionary) uncovered fewer than 600 fully-nested five-letter words (ones containing all four-letter and three-letter words, such as SPARK: SPAR, PARK, SPA, PAR, ARK), and she was unable to construct a legitimate word square from this stockpile. In view of the experimental support value of 455 calculated in "How Many Words Support a Square" in the May 1992 **Word Ways**, it appears that she was somewhat unlucky. Or, it is entirely possible that her stockpile did contain a legitimate five-by-five square, but, working by hand, she failed to discover it.

If one enlarges the stockpile, the problem becomes easy, at least for the single five-by-five word square. We used a database of 15,719 five-letter words, containing all the Webster's Second words in the Air Force Dictionary, all the words in the Official

Scrabble Players Dictionary (1978), and Chambers Words (1985), plus some from the Oxford English Dictionary and Webster's Third. Testing them against 8,072 four-letter and 1,707 three-letter words, we found 1,244 of them to be fully nested. From these, the computer constructed 12,696 single five-by-five and 5,932 double ones. Interestingly, the support calculated from this data suggests that 188 randomly-chosen words from the 1,244-word stockpile ought to be enough to construct, on the average, a single five-by-five square; the corresponding support for a double square is 522. Fully-nested words are peculiarly amenable to square construction, making Motamen's failure to find one even more inexplicable.

A single word square with all nested squares single is, of necessity, a progressive word square (one in which each word in turn discards the first letter of the preceding word and adds a new letter at the end). Progressive word squares can be succinctly characterized by a sequence of nine letters: the five letters of the word in the top row, followed by the terminal letters of the second through fifth words. This is the basis for the summary of the 147 single word squares with all nested single squares, listed below. In each letter array, every sequence of nine letters represents a square. Read the array from left to right; when additional rows are encountered, one may follow either one (thus, BRASEREST, BRASHEDER and BRASHETHE are all word squares). Any of the parenthesized letters or letter-groups may be appended to the start of a sequence (thus, BASHEREST and TRASHERES are both squares). The numerical indicator 37(13) says that there are 37 squares in the corresponding array, and the longest sequence of letters in the array is 13. Certain arrays intersect. The first five arrays share the word square HETHEREST; the first four arrays also share the squares SHETHERES and ASHETHERE; the first two arrays also share the word square BASHETHER; the sixth and seventh arrays share AREASERES. Collectively, these word squares form four directed networks with no internal loops; the main network consists of all arrays but the final three. For details on directed networks, see "A Word String Network" in May 1991, and the three-part "Directed Word Chain Networks" in May and August 1991.

```
(B,C,D,F,G,L,M,P,TR,W)ASHEREST        37(13)
                        THENS
                           REST
            ST
(B,C,S)ABASERER                       30(14)
          HEDER
            THEREST
            RD
             EST
             K

BLASEREST                              9(13)
    HEDER
      REST
      THEREST
```

THEREST 11(13)
BRASEREST
HEDER
RE ST
THEREST

(AS,B,H,M,T,W)ETHEREST 7(10)

(ARE,ER,PE,SC,TE,URE)ASERER 7(10)
ST

STAREASEREST 9(11)
REST
DD
E

(C,B,M,W)OTHEREST 4(9)

(B,CL,E,F,M,N,R)ATHEREST 8(10)

THEREST 9(11)
(F,P)LEASERER
ST

TERASENES 4(10)
RER
ST

(BUSH,FLAM,GRAP,SCAP,SHOT,STAP,STRAP)EREST 8(10)

ASETHEREST 2(10)

(G,N,P,S,T,W)ALASTYED 12(9)
S

(B,P)USHEREST 2(9)

What about nested squares in a double word square? The 5,932 double squares contain a wide variety of nested squares within them: single, double, hybrid, defective. None of the double five-by-five squares contains nine double three-by-three squares, but there are probably several hundred with eight double three-by-three squares, with the ninth either single or hybrid. It is hard to count how many there are because the fault can occur in many different places; however, by choosing to accept defects in this or that place and putting appropriate instructions into the computer program, one can find squares like the ones illustrated below. In the first two pairs, the hybrid three-by-three is part of a hybrid four-by-four; in the last square, there is one single three-by-three in addition to eight doubles, and the four-by-four squares are all double ones.

N A S T Y	B A S T A	S T A W N	B L A D E
O t h e r	O t h e r	W a S H E	R E v e r
W h O R E	W h O R E	a r e a R	A W e n e
N e W E N	N e W E N	D e E S E	N E r e s
E r E S T	E r E S T	S a L E S	T R A S T

What are the prospects for extending this study to nested six-by-six word squares? It seems unlikely that any can be found

within the confines of Webster's Second, Webster's Third and the Oxford English Dictionary. If one tests the 1244 fully-nested five-letter words for overlapping crashes (that is, form letter-sequence ABCDEF from fully-nested words ABCDE and BCDEF) and determines which of these are in fact six-letter words, one finds 230 of them, listed below. These form the raw material for a six-by-six nested square, but, judging from the support statistics calculated for five-by-five nested squares, it is clear that a considerably larger set is needed.

abaser	brathe	croose	graped	mother	rooses	souses	tether
abashe	busher	dashed	grapes	mouses	rowers	sowers	thawed
agates	cagers	dasher	grates	mowers	sagest	spales	thawer
agenes	carene	dashes	greese	musers	scales	spanes	themes
amates	carere	earest	groped	nather	scaped	spared	theres
amides	carest	eather	gropes	neared	scaper	sparer	thouse
amused	cashed	eraser	groups	nevels	scapes	spares	totems
amuser	casher	fashed	grouts	nooses	scared	spates	towers
amuses	cashes	fasher	haired	paired	scarer	staker	traped
ananas	catell	fashes	hewers	palays	scares	stakes	trapes
areare	chaine	father	houses	papern	seared	stales	urease
arears	chains	ferest	laches	parent	seathe	stapes	vowers
asethe	chaire	flaked	lament	pashed	serest	stared	wagers
atokes	chapes	flaker	lashed	pashes	sethen	starer	warent
aweels	chared	flakes	lasher	peares	sewers	stares	warest
baches	chares	flamen	lashes	please	shales	strade	washed
barest	chawed	flamer	lather	powers	shapes	stramp	washer
bashed	chawer	flanes	leared	prosed	shared	strays	washes
basher	chides	flemes	leares	proses	shares	swaged	weared
bashes	chores	forest	leaser	pusher	shawed	swager	wethen
bather	choses	gagers	levels	raches	sheder	swages	wether
blasty	chouse	galays	lewest	ragers	shewel	taches	whares
booses	chouts	gamene	looses	rasher	shewer	tavers	wheels
bother	cleare	garest	lowers	rashes	shores	tavert	wheres
bowers	clears	gashed	mashed	rather	shouts	teared	wholes
braked	cleath	gasher	masher	reared	shower	teaser	whoops
braker	copens	gashes	mashes	revels	shules	teathe	whores
brakes	cowers	godsos	merest	revere	skates	tepees	wother
brasen	croons	gooses	mether	revert	sorest		