

PACKING WORDS

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What is the minimum rectangular area in which one can pack precisely N words of length L ? All must be joined in criss-cross fashion, and no words of other lengths are allowed. The main point is to find the packing patterns; finding real (and, if possible, common) words to fit those patterns adds interest, converting a purely mathematical problem to a logological one as well. The patterns derived here may be used in conventional crossword or other puzzles.

Packing patterns follow recognizable sequences as N increases. There are two types, depending on whether L is odd or even. This article describes packing four-letter and five-letter words; plans for six-letter or seven-letter words can be easily derived from them. I do not give a full scheme for six-letter words, but exhibit one large arrangement. For all word sizes, the examples given here are essentially portions of ordinary double word squares strung together.

Consider first four-letter words ($L=4$). In the following examples there is a fundamental pattern for five words ($N=5$). This is not the best possible packing, but it is the best which can be logically extended to larger values of N . Adding blocks extends it to 12, 19, 26 (not shown), etc. There is another fundamental pattern for $N=6$, to which one can add a word at a time to allow packing 7 or 8. For $N=9$, one must start to build a new block.

N	M	A	I	N	N	M	A	I	N	M	A	I	N	M	A	I	N	M	A	I	N
O	E	O	O	V	E	R	O	E	O	O	V	E	R	O	E	O	O	E	O		
N	A	N	N	A	M	E	N	A	N	N	A	M	E	N	A	M	E	N	A	N	
E	A	T	S	E	E	A	T	S	E	E	A	T	S	E	E	A	T	S	E	E	
5					12					19											

M	A	I	N	N	M	A	I	N	T	E	X	I	T	T	E	X	I	T						
O	V	E	R	O	V	E	R	O	H	E	A	R	H	H	E	A	R	H	E	A	R	H		
A	M	E	N	N	A	M	E	N	E	R	A	R	E	E	R	A	R	E						
E	A	T	S	E	A	T	S	E	M	A	N	Y	M	A	N	Y	M	A	N	Y	M	A	N	Y
6				7				8				9				10								

T	E	X	I	T	E				E	X	I	T	E	X	I	T	E	X	I	T	E	X	I	T
H	E	A	R	H	E	A	R	H	H	E	A	R	H	H	E	A	R	H	H	E	A	R	H	H
E	R	A	R	E	R	A	R	E	R	A	R	E	R	A	R	E	R	A	R	E	R	A	R	E
M	A	N	Y	M	A	N	Y	M	A	N	Y	M	A	N	Y	M	A	N	Y	M	A	N	Y	M
11					13					15														

All of the above (Set A) are four units high. The patterns on the next page (Set B) are five units high.

R	F	R	R	F	R	R	R	F
BALL	BALL	BALL	BALL	BALL	BALL	BALL		
AREA	AREA	AREA	AREA	AREA	AREA	AREA		
NEST	NEST	NEST	NEST	NEST	NEST	NEST		
D	S	D	SEND	S	D	SEND	SEND	S
7		15			23			

RAFT	R	RAFT	R	R	RAFT
BALL	BALL	BALL	BALL	BALL	BALL
AREA	AREA	AREA	AREA	AREA	AREA
NEST	NEST	NEST	NEST	NEST	NEST
D	D	SEND	D	SEND	SEND
8		16		24	

R	R	R	R	R	R			
BALL	B	BALL	BALL	B	BALL	BALL	BALL	B
AREA	A	AREA	AREA	A	AREA	AREA	AREA	A
NEST	N	NEST	NEST	N	NEST	NEST	NEST	N
D	SEND	D	SEND	SEND	D	SEND	SEND	SEND
9		17		25				

The table below summarizes the areas needed to pack N four-letter words for either Set A or Set B; the final line lists the smallest area allowed by any packing. Packing 31 words in 5x19 is the last case in which a five-high frame is useful. There are three special cases. Eight words pack in a normal 4x4 square; the scheme for seven words in Set B can be rotated into a four-high pattern; and for 26 words, packing in a 9x9 square (described below) beats either set. The final sequence could never serve as an IQ test unless you see how it was made!

words	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
areaA	4	16	16	16	24	24	(24)	24	24	36	36	44	44	44	44	56	56	56	64	64	64	64	76
areaB							20	25	30						45	50	55						70
best	4	16	16	16	24	24	20	(16)	30	36	36	44	44	44	44	50	55	56	64	64	64	64	70

words	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
areaA	76	76	84	84	84	84	96	96	96	104	104	104	104	116	116	116	124	124
areaB	75	80					95	100	110							120	125	130
best	75	76	(81)	84	84	84	96	95	96	104	104	104	104	116	116	116	124	124

Frames more than five units high usually result in a loss of packing efficiency, but one can get neat constructions in square envelopes. The word pattern in the 14x14 envelope at the top of the next page is made by the use of two basic units, a solid and a hollow block. The 9x9 array to the right uses the same blocks in a similar pattern; it contains 24 words. The third array is less symmetrical but contains 26 words. As already noted, this uses three fewer cells to pack 26 words than does the Area A construction.

All diagrams shown in this article use common words. The 4x4 square on the next page appears ordinary, but it is special in that it allows extending the central words in all directions into eight other words. By going to the computer for a special search, one can find squares with wrap-around properties that allow one square to be used throughout an array. Limiting the database

SCAB SITE SCAB	SCAB SITE	SCAB SIGN	SCAB
H RENT ACHE E	H RENT A	H RENT R	HERE
IDEA E S IDEA	IDEA E S	IDEA EDIT	IDEA
PEAR PRAY PEAR	PEAR PRAY	PEAR WINO	PEAR
L A S	L J	L S M	
SITE SCAB SITE	SITE SCAB	SITE SCAB	
T ACHE E T A	T A HERE	I ITCH R	
E S I EASE S	E SAID A	ZINC IDEA	
PRAY PEAR PRAY	PRAY PEAR	E SHOP A	
A S J	24	26	
SCAB SITE SCAB			
HERE T A HERE			
I EASE SAID A			
PEAR PRAY PEAR			

to 2500 common words allowed only a few squares like the ones below. The wrap-around sequences ALAREAL, RESTIRE, ALTORAL and MAIMAMA are reminiscent of directed word chains.

CAME CAME CAME	BOSS BOSS	CAME BOSS
A AREA REAL R	U TOFU O	ALAR UNTO
STIR S R STIR	STEW S W	STIR STEW
TOMS TOMS TOMS	TOMS TOMS	TOMS TOMS
R A A	R U	
CAME CAME CAME	BOSS BOSS	
A R A AREA R	U O UNTO	
S REST R S R	S WEST W	
TOMS TOMS TOMS	TOMS TOMS	
R R A	24	
CAME CAME CAME		
ALAR A R ALAR		
S IRES REST R		
TOMS TOMS TOMS		

Various plans for packing five-letter words are used in the following. One word can be removed at a time from the 15-word array to allow 14, 13, 12 or 11 words. No smaller rectangle containing precisely 11 or 10 words is possible. One or two words can be removed from the 19-word array, but three words cannot, so packing 16 five-letter words requires inefficient appending to the 15-word array.

MAPLE	MAPLE M	MAPLE	MAPLE MAPLE
A H N	A H N A	ASHEN	A HENNA H N
SCOPE	S OPENS	SCOPE	SCOPE SCOPE
T T M	T T M T	TOTEM	TOTEM M T M
STORY	STORY S	STORY	STORY STORY
6	7	10	15

CLIMB	CLIMB C	CLIMB CLIMB C
HONOR	H NORTH	H NORTH NORTH
AWARE	AWARE A	AWARE AWARE A
RENEW	R NEWER	R NEWER NEWER
TRESS	TRESS T	TRESS TRESS T

Just as four-letter words can be packed in frames five units high, so five-letter words can be packed in frames six units high. As before, determination of when L+1 is better than L is a matter of arithmetic. A summary is given in the table below. There are two exceptions (in parentheses), the normal 5x5 square and a case (described below) where a large square array allows an improved packing.

A B	A	A B	A	A	A B
ABOUT	ABOUT	ABOUT	ABOUT	ABOUT	ABOUT
FORTH	FORTH	FORTH	FORTH	FORTH	FORTH
ORATE	ORATE	ORATE	ORATE	ORATE	ORATE
OTTER	OTTER	OTTER	OTTER	OTTER	OTTER
T E E	T E R E C T	E E	T E R E C T	E R E C T	E E
9	19		29		

GAILY	A	CADGE	E	S	ABHOR
GARNI	ABACK	CABAL	DACHA	FACES	ABLER
RUSES	BORON	AROMA	ETHOS	ATOLL	ROUND
ADEPT	BUGLE	BOOES	FEINT	KYRIE	DUNCE
CYSTS	ETUDE	BLESS	ENVOI	IRATE	OTTER
E	Y E S S A Y		R E R R O R	L E P E R	
10	20		30		

Y	GOATS	Y	Y	GOATS
FABLE	G U E	FABLE	FABLE	G U E
ACRID	ULTRA	ACRID	ACRID	ULTRA
THING	S E T	THING	THING	S E T
STAGE	TYRES	STAGE	STAGE	TYRES
O RODEO		O RODEO	RODEO	
16		26		

Y		Y	Y	
FABLE	F B E	FABLE	FABLE	F B E
ACRID	ACRID	ACRID	ACRID	ACRID
THING	T I G	THING	THING	T I G
STAGE	STAGE	STAGE	STAGE	STAGE
O RODEO	R D	O RODEO	RODEO	R D
15		25		

words	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
areaA	35	35	35	55	55	55	55	55	75	65	65	65	85	85	85	85	85	100	95	95	95
areaB		30	36					66	72			66	72					102	108		
best	35	30(25)	55	55	55	55	55	55	72	65	65	65	72	85	85	85	85	100	95	95	95

words	29	30	31	32	33	34	35	36	37	38	39	40
areaA	115	115	115	115	115	135	125	125	125	145	145	145
areaB	102	108				138	144			138	144	
best	102	108	115	115	115(121)	125	125	125	145	138	144	

Packing in large squares is more efficient for five-letter words than for four-letter words. Two criss-cross patterns are illustrated on the next page. A 5x5 double word square places ten words in 25 cells for 0.400 words per cell; the two squares illustrated pack at 0.282 words per cell. They were developed by joining 5x5 squares found by computer; in some places the reader can recognize the original square by inserting the letter(s)

deleted in order to make the joins.

CABLE	KVASS	OMENS	KNACK	FARAD
A LOWER	V LEVEL	M	N TROLL	A A
SLAVE	ALIBI	ETUDE	IGLOO	EAVED
EASER	ALARM	NADIR	F ANKLE	E D
DRESS	LANES	SLEEK	EASES	TODAY
G	N A	T	V	P R
DEALS	GOADS	FEAST	LASTS	HEWED
A RIATA	U UHLAN	R	ESTOP	ARRAY
WHETS	UNDER	ATONE	ATONE	RAISE
N NESTS	I FAMED	A	S RENTS	T R
SCARY	SPOTS	ERECT	TRYST	HEEDS
O	A H	O		
SPORT	ASPIC	GNOME		
LEMUR	PHONO	RUBES		
ENEMY	RAKED	ADITS		
W GESSO	E ADMIT	E		
SLANT	NERDS	PESOS		

34

82

Packing six-letters words follows the general plan used for four-letter ones; the reader may enjoy working out the arithmetic. In the six-letter word square below, packing efficiency approaches that of ordinary crosswords. Packing seven-letter words is even more efficient, but finding actual words is difficult; BASIC is too slow for finding 7x7 double squares. I leave the task of filling in the array below to programmers like Ted Clarke.

CABALA	EAGLES	DABBED	A A A A A A A	A A A A A A A	A A A A A A A
A ACUMEN	I PREVUE	U	A A A A A A A A	A A A A A A A A	A A A A A A A A
PERUSE	TORQUE	FERRIC	A A A A A A A	A A A A A A A	A A A A A A A
OLEATE	A U A	ARDENT	A A A A A A A	A A A A A A A	A A A A A A A
TASTER	I O R	TSETSE	A A A A A A A	A A A A A A A	A A A A A A A
ENTERS	LITRES	SENSED	A A A A A A A	A A A A A A A	A A A A A A A
D	L	C	A A A A A A A	A A A A A A A	A A A A A A A
ISLAND	BIASED	SWEATY	A	A A A	A
M I E E	CAVORT	M A	A A A A A A A	A A A A A A A	A A A A A A A
A N J	ABATED	EYEBAR	A A A A A A A A	A A A A A A A A	A A A A A A A A
GREASE	RACING	R U R	A A A A A A A	A A A A A A A	A A A A A A A
E A	CRETIN	E I S O	A A A A A A A A	A A A A A A A A	A A A A A A A A
SPROUT	ROASTS	CASHEW	A A A A A A A	A A A A A A A	A A A A A A A
I	N	V	A A A A A A A A	A A A A A A A A	A A A A A A A A
MATTER	PSALMS	FABLED	A A A A A A A	A A A A A A A	A A A A A A A
ANYONE	I L H	IGUANA	A	A A A	A
COMPTS	R E I	LETTER	A A A A A A A	A A A A A A A	A A A A A A A
ASPIRE	AUXINS	INTERN	A A A A A A A	A A A A A A A	A A A A A A A
W ACETIC	I TRADES	E	A A A A A A A	A A A A A A A	A A A A A A A
SENSES	YEARNS	LASTED	A A A A A A A	A A A A A A A	A A A A A A A
			A A A A A A A	A A A A A A A	A A A A A A A
			A A A A A A A A	A A A A A A A A	A A A A A A A A
			A A A A A A A	A A A A A A A	A A A A A A A