INCOMPATIBLE STRINGS

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The following chessboard pattern is a template for the words ONE to NINE:

	W	T	
	O	U	H
F	N	G	R
X	I	E	
	S	V	

Starting with the letter O and stepping from square to square as a king moves in chess, we can reach N and then E, to spell ONE. Restarting with T we can spell TWO, and so on for all the cardinals up to NINE. ZERO won't go, but NOUGHT will. Note that no letter on the template is repeated. For the doubled E in THREE the "move" remains on the same square. Some natural questions prompted by the template are:

- Can a template be found that will also accommodate ZERO and/or TEN, etc.?
- If we start with a different number, can a template be found to generate a longer list of consecutive cardinals?
- What template will generate the greatest total of not necessarily consecutive cardinals?
- Can templates be found to generate other interesting groups of related words?

Ross Eckler aroused my interest in this topic by asking if I could prove that a template can always be found to accommodate **any** pair of six-letter words, as trials seemed to suggest. Change **words** to **strings** and this is a mathematical question. Using digits as variables for letters, after some experiment I found a pair of isograms (every sign appears once), 123456 and 153624, that cannot share the same template, an **incompatible pair.** In such a template 2 and 5 would both have to abut 1, 3, 4, and 6, while 3 would have to abut 4 and 6. Trial will show this to be impossible. The next task was to find a pair of words to match these string structures. Using my PC to run through a lexicon stored on disk, I found two: PEARLY-PLAYER and PAINTS-PTISAN. Ross's conjecture was thus proved wrong.

Now if a word can be spelled on a template then so can its reverse, and if a word cannot be spelled on a template then its reverse cannot be either. Hence if 123456-153624 is an incompatible pair, then so is 123456-426351. Two word pairs that fit this reversed pattern are LISTEN-TINSEL and SAILED-LADIES. Note however that although 123456-153624 maps onto both PEARLY-PLAYER and PLAYER-PEARLY, 123456-426351 maps onto LISTEN-TINSEL only, while TINSEL-LISTEN maps onto 123456-624153. This seems strange, but follows from the fact that the structure of any pair can be represented in two ways, depending upon which string is identified with 123456, the reference standard. Thus the structure of 123456-426351 is the same as that of 624153-123456. In some cases, however, the letter swaps involved in going from one string to the other are symmetrical, which means that the two representations turn out identical: 123456-153624 is the same as 152624-123456.

Are there any more 6-letter incompatible pairs? A computer program I wrote scanned all 6!=720 permutations of the six digits and tested each one against every one of the templates that will accommodate the string 123456. The latter were generated by a routine that starts with 1 in the centre cell of a two-dimensional array, and then runs through all the possible ways to place 2,3,4, 5,6 in adjacent cells, moving king-wise on each step. Permutations that failed on every template are incompatible with 123456, but half of them are equivalents of the other half in the sense explained above. The 13 cases found are shown below. Those word pairs I have found that satisfy these patterns are seen on the right. Nine gaps remain to be filled.

1	123456-146253 = 146253-123456		
2	123456-153624 = 153624-123456	PEARLY-PLAYER, PAlNTS-PTISAN	
3	123456-253614 = 513624-123456		
4	123456-254163 = 416325-123456		
5	123456-264153 = 416352-123456	UNABLE-NEBULA	
6	123456-351426 = 351426-123456	ASPIRE-PRAISE, FLUENT-UNFELT	
7	123456-351462 = 361425-123456		
8	123456-352641 = 631524-123456		
9	123456-361452 = 361452-123456		
10	123456-425136 = 425136-123456		
11	123456-426315 = 524163-123456		
12	123456-426351 = 624153-123456	LISTEN-TINSEL, SAILED-LADIES	
13	123456-523614 = 523614-123456		