AN ABELIAN ALPHABET?

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A curious problem may be stated very simply for those familiar with the language of mathematics. Consider the free group on the 26 letters of the alphabet, with transposals as relations; is the group Abelian?

What this amounts to for the non-mathematician is the following. Words are considered as strings of letters, as is so often the custom in Word Ways, and we say two words are equivalent if they are transposals of each other. For example, EAR=ERA, RACE=ACRE, and UNITE=UNTIE. In such an equation, or relation, between two words, we are allowed a simple cancelling process. If both words have the same initial letter, or if both words have the same final letter, we can cancel it on both sides. For example, EAR=ERA results in AR=RA. RACE=ACRE in RAC=ACR, and UNITE=UNTIE in NITE=NTIE.

The result is obviously an equation between strings of letters only, since in general the cancelling process rarely results in words. Applying the cancelling process twice more to NITE=NTIE results in IT=TI. The ultimate goal is to provide sufficiently many transposal pairs to conclude that xy=yx, where x and y represent any two distinct letters of the alphabet. For example, we already have RA=AR and IT=TI from the examples above. In the jargon of mathematics, xy=yx is described by saying that x and y commute.

It is reasonably clear that it will prove well-nigh impossible to show, for instance that QZ=ZQ by the same technique as that of the example UNITE=UNTIE, since this would necessitate finding two transposals, one containing the digram QZ, the other QZ, as well as both words containing the same cancellable letters. This difficulty is overcome by simplifying an equation using any equations between letter strings that have already been derived. For example, we have seen that RACE=ACRE results in RAC=ACR, but we also know that AR=RA, and it follows that ARC=ACR, so that ARC=RAC=ACR. From ARC=ACR now follows RC=CR on cancelling the initial A. Another example: CARE=RACE, so CAR=RAC (cancellation of final E), so CRA=RAC (using AR=RA), so RCA=RAC (using CR=RC), so CA=AC (cancellation of initial R). It is essential when cancelling that it be performed only on the leftmost, or rightmost, letters. From CAR=RAC It is impossible to deduce immediately that CA=AC; the auxiliary information given by RA=AR, RC=CR is needed.

Once the process has been mastered, it is quite easy to reduce an equation between two transposals. It is not too difficult, and a good exercise, to show first of all that each of the vowels AEIOU commutes with every other letter of the alphabet. This then means that in any future transposal pair, the vowels can be ig-
nored. For example, QUINZE-ZEQUIN would lead to the equation UQUINZE=ZEUQIN=UZEQIN using the fact that U commutes with Q,E,Z. Cancellation then gives QUINZE=ZEQUIN. Similar reduction with Q and E leads to QNZ=ZNQ; however, further information is required before QZ=ZQ can be deduced.

To show that x and y commute, a transposal pair must be found in one word of which x precedes y, and in the other, y precedes x. This is where the difficulty begins, because of the intractable letters J,Q,X,Z. If the whole problem is to be solved, then it seems that recourse to large dictionaries must be allowed. On aesthetic grounds, however, hyphenated words (e.g. SIXTY-SEVEN=SEVENTY-SIX) should be avoided. On a personal whim, I have also excluded two-letter words.

In practice, I have found it best to draw a 26x26 grid, filling in the xy square (and the yx square) when x and y have been shown to commute. This allows seeing immediately the letter pairs that already commute, which helps in writing down suitable further transposal pairs. The following transposals have been taken from Chambers Twentieth Century Dictionary, the Merriam-Webster Unabridged dictionaries (Second and Third Editions), Funk & Wagnalls Unabridged, and the Oxford English Dictionary (asterisked words are under a different head-word therein). The list shows that every possible pair of letters commutes, with the exception of JX, KQ, QW, QX, VX and XZ. The listing of bigrams follows a specific nonalphabetic order: each letter pair can be shown to commute using transposals earlier in the table. The choice of transposals is, of course, arbitrary; there are many other possibilities for the common letters.

To complete the problem of showing that the alphabet is Abelian, it is necessary to find transposal pairs for the six missing cases. I look forward to seeing in Colloquy examples found by readers!

There are several variations on the original problem. For example, within the confines of a relatively small dictionary, is it possible to show that just the vowels AEIOU commute with every other letter?

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AB abba=baba AC act=cat AD add=dad AE tae=tea AF aft=fat AG agin=gain AH tahr=char AI dai=dai AJ ajwan=kain AL alp=lal AM amp=map AN any=nay AO gaol=goal AP apt=pat AR ear=era AS asp=sap AT eat=eta AU gaur=guar AQ aqua=qua AV ave=vae AM awe=wa AX coax=coxa AY yae=yae AZ azo=zoa BC bac=cab BD bad=dab BE beer=eber BG bag=gab BK bak=kab BL bal=lab BM bam=mab BN ban=nab BO boe=obe BP bap=pab BR bar=rab BS bas=sab BT bat=tab BW bab=wab CD cade::dace CF cafe::face CJ caja=jaca CL cal=lac CM cam=mac CN cane=nace CP cap=pac CR cark=rack CS cask=sack CT cate::tace CU caul=ucal CV cavate:vacate CW cawk=wack DE deify::edify DG dag=gad DH dah=dad DI ide=die DK daku=kadu DL dal=lad DM dam=mad DN dander=nadder DO ado=oda DP dap=pad DR dar=rad DS das=sad DT date::tade DU duode=udo DV daver=vader DW daw=dad DY day=yad EF lief=life EG egal=geal EH seah=shea EI le=lie EL elt=let EM eman=mean EN enam=nas EO leo=loe EP rape::reap ER are=era ES esse=esse ET ate=eat EU leu=lue EV ever=ever EW ewe=ewe EX exam=xema EY eyed=yeed EZ ezel=zeel FO fot=oft FR fart=raft FY fay=yaf GJ gaj=jag
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