AN ABELIAN ALPHABET?

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A curious problem may be stated very simply for those familiar with the language of mathematics. Consider the free group on the 26 letters of the alphabet, with transposals as relations; is the group Abelian?

What this amounts to for the non-mathematician is the following. Words are considered as strings of letters, as is so often the custom in **Word Ways**, and we say two words are **equivalent** if they are transposals of each other. For example, EAR=ERA, RACE=ACRE, and UNITE=UNTIE. In such an equation, or relation, between two words, we are allowed a simple cancelling process. If both words have the same initial letter, or if both words have the same final letter, we can cancel it on both sides. For example, EAR=ERA results in AR=RA, RACE=ACRE in RAC=ACR, and UNITE=UNTIE in NITE=NTIE.

The result is obviously an equation between strings of letters only, since in general the cancelling process rarely results in words. Applying the cancelling process twice more to NITE=NTIE results in IT=TI. The ultimate goal is to provide sufficiently many transposal pairs to conclude that xy=yx, where x and y represent any two distinct letters of the alphabet. For example, we already have RA=AR and IT=TI from the examples above. In the jargon of mathematics, xy=yx is described by saying that x and y commute.

It is reasonably clear that it will prove well-nigh impossible to show, for instance that QZ=ZQ by the same technique as that of the example UNITE=UNTIE, since this would necessitate finding two transposals, one containing the digram QZ, the other ZQ, as well as both words containing the same cancellable letters. This difficulty is overcome by simplifying an equation using any equations between letter strings that have already been derived. For example, we have seen that RACE=ACRE results in RAC=ACR, but we also know that AR=RA, and it follows that ARC=RAC, so that ARC=RAC=ACR. From ARC=ACR now follows RC=CR on cancelling the initial A. Another example: CARE=RACE, so CAR=RAC (cancellation of final E), so CRA=RAC (using AR=RA), so RCA=RAC (using CR=RC), so CA=AC (cancellation of initial R). It is essential when cancelling that it be performed only on the leftmost, or rightmost, letters. From CAR=RAC it is impossible to deduce immediately that CA=AC; the auxiliary information given by RA=AR, RC=CR is needed.

Once the process has been mastered, it is quite easy to reduce an equation between two transposals. It is not too difficult, and a good exercise, to show first of all that each of the vowels AEIOU commutes with every other letter of the alphabet. This then means that in any future transposal pair, the vowels can be ignored. For example, QUINZE=ZEQUIN would lead to the equation UQINZE=ZEUQIN=ZUEQIN=UZEQIN using the fact that U commutes with Q,E,Z. Cancellation then gives QINZE=ZEQIN. Similar reduction with E and I leads to QNZ=ZQN; however, further information is required before QZ=ZQ can be deduced.

To show that x and y commute, a transposal pair must be found in one word of which x precedes y, and in the other, y precedes x. This is where the difficulty begins, because of the intractable letters J,Q,X,Z. If the whole problem is to be solved, then it seems that recourse to large dictionaries must be allowed. On aesthetic grounds, however, hyphenated words (e.g. SIXTY-SEVEN=SEVENTY-SIX) should be avoided. On a personal whim, I have also excluded two-letter words.

In practice, I have found it best to draw a 26x26 grid, filling in the xy square (and the yx square) when x and y have been shown to commute. This allows seeing immediately the letter pairs that already commute, which helps in writing down suitable further transposal pairs. The following transposals have been taken from Chambers Twentieth Century Dictionary, the Merriam-Webster Unabridged dictionaries (Second and Third Editions), Funk & Wagnalls Unabridged, and the Oxford English Dictionary (asterisked words are under a different head-word therein). The list shows that every possible pair of letters commutes, with the exception of JX, KQ, QW, QX, VX and XZ. The listing of bigrams follows a specific nonalphabetic order: each letter pair can be shown to commute using transposals earlier in the table. The choice of transposals is, of course, arbitrary; there are many other possibilities for the common letters.

To complete the problem of showing that the alphabet is Abelian, it is necessary to find transposal pairs for the six missing cases. I look forward to seeing in Colloquy examples found by readers!

There are several variations on the original problem. For example, within the confines of a relatively small dictionary, is it possible to show that just the vowels AEIOU commute with every other letter?

AB abba=baba AC act=cat AD add=dad AE tae=tea AF aft=fat AG agin=gain
AH tahr=thar AI dail=dial AJ ajwan=jawan AK akin=kain AL alp=lap
AM amp=map AN any=nay AO gaol=goal AP apt=pat AR ear=era AS asp=sap
AT eat=eta AU gaur=guar AQ aqua=quaa AV ave=vae AW awe=wae AX coax=coxa
AY aye=yae AZ azo=zoa BC bac=cab BD bad=dab BE beer=eber BG bag=gab
BK bak=kab BL bal=lab BM bam=mab BN ban=nab BO boe=obe BP bap=pab
BR bar=rab BS bas=sab BT bat=tab BW baw=wab CD cade=dace CE cetene=ectene
CF cafe=face CJ caja=jaca CL cal=lac CM cam=mac CN cane=nace CP cap=pac
CR cark=rack CS cask=sack CT cate=tace CU caul=ucal CV cavate=vacate
CW cawk=wack DE deify=edify DG dag=gad DH dah=had DI ide=die DK daku=kadu
DL dal=lad DM dam=mad DN dander=nadder DO ado=oda DP dap=pad DR dar=rad
DS das=sad DT date=tade DU duo=udo DV daver=vader DW daw=wad DY day=yad
EF lief=life EG egal=geal EH seah=shea EI lei=lie EK eek=eke EL elt=let
EM emane=meane EN ean=nae EO leo=loe EP rape=reap ER are=era
ES esse=sese ET ate=eat EU leu=lue EV ever=veer EW ewe=wee EX exam=xema
EY eyed=yeed EZ ezel=zeel FO fot=oft FR fart=raft FY fay=yaf GJ gaj=jag

GL gal=lag GM gam=mag GN gan=nag GO goam=ogam GP gape=page GR gar=rag
GS gas=sag GT gat=tag GU guly=ugly GV gave=vage GW gaw=wag GZ gaz=zag
HJ haj=jah HK haka=kaha HL hal=lah HM ham=mah HN han=nah HO pho=poh
HP hap=pah HR har=rah HS ash=has HU hullo=uhllo HW haw=wah HY hay=yah
IL ilka=laik IM imam=maim IN ins=nis IO lion=loin IR air=ria IS isle=sile
JR jar=raj JT jat=taj KL kale=lake KM kame=make KO koa=oak KR kari=raki
KT kat=take KU aku=auk KV kavass=vakass KW kawa=waka KY kay=yak
LM lam=mal LN lana=nala LO lope=olpe LP lap=pal LT late=tale
LU glue=gule LV lave=vale LX axl=lax LY lay=yal MN man=nam MO moit=omit
MP map=pam MR mar=ram MS mas=sam MT mat=tam MU muset=umset MW mawe=wame
MY may=yam MZ maza=zama NO nous=onus NP nap=pan NR nar=ran NS nas=san
NT nat=tan NU gnu=gun NV nave=vane NW naw=wan NY nay=yan OP opt=pot
OR ort=rot OS ose=soe OT otter=toter OU moun=muon OV ovile=voile
OW owe=woe OY oye=yoe OZ ozonic=zoonic JP japonism=pajonism PR par=rap
PS pas=spa PT pat=tap PU pus=ups PW paw=wap PY pay=yap RS ras=sar
RT rat=tar RU rue=ure RV rave=vare RW raw=war RY ray=yar ST star=tsar
SU sue=use SV save=vase SW saw=was SY say=yas TU tau=uta TW taw=wat
TY tay=yat UV uva=vau WY way=yaw IT mite=time BI bit=tib CI cima=mica
FI fire=rife GI git=tig HT het=the HI whit=with IK kite=tike IP hip=phi
IV live=vile LR leer=reel LS lose=sole KN ken=nek CH chi=hic
CK hackin=kachin FK fakir=kafir CG cierge=griece CO canoe=ocean
BF brief=fibre BU bun=nub BH bush=hubs BV observe=verbose BY yerba=barye
DF deaf=fade FN fen=nef FS fist=sift FL file=life FH flesh=shelf
FT feathers=tashreef LW lewd=weld FW flow=wolf FG flog=golf FU furs=surf
FM flemish=himself FP forpit=profit GK gink=king GH charge=creagh
UY puya=yaup CY cuya=yuca GY guy=yug HV chave=vache VY vying=yngvi
IU inured=ruined IW sinew=winse NZ nazi=zain IZ niz=zin KP keep=peek
KS inks=skin JS jamboks=sjambok JM jamas=samaj MV mover=vomer
LZ laze=zeal TV tove=vote UW unware=wauner VW vowels=wolves JO ajos=soja
BJ bojo=jobo DJ dojo=jodo JK jauk=kaju EJ joiner=rejoin JU ujis=juis
JQ jaques=quasje JN gunj=jung IJ jaina=inaja JL lija=jail FJ jiff=fijf*
JV javar=vajra JW jewis=wijse* JY enjoy=yojen JZ jiz=zij FV fyve*=vyfe
IY miry=yirm PV overpass=passover QS quads=squad EQ equals=squeal
QR quester=request LQ equals=lasque IQ quires=risque MQ masque=squeam
QU qus=suq PQ equip=pique QT qualite=tequila OQ quote=toque
FQ fique=queif BQ basque=quebas NQ enquiet=quieten CQ cinque=quince
HQ cheque=queech DQ dequeen=queened GQ gasquine=queasing
QY quyty*=yquyt RZ razed=zerda BZ braze=zebra DZ adzed=zedad
HZ hazel=zelah CZ cloze=zocle UZ chuze=zuche QZ quinze=zequin FZ fiz=zif
PZ phiz=ziph WZ wiz=ziw SZ suez=zeus TZ nietzsche=zechstein
HX exhall*=hallex DX detax=taxed RX retax=taxer UX exul=ulex
SX nexus=unsex TX exalt=latex IX sixty=xysti XY xyst=styx CX cylix=xylic
NX unwax=waxun* OX axon=noxa WX woxyn*=xowyn BX boxer=exorb
FX foxier=xerifo* YZ phytozoa=zoophyta KZ kozos=zooks PX expos=poxes
MX exclaim=mexical GX taxing=xangti KX boxwork=workbox QV queven=venque
VZ veze=zeve