INTRODUCTION TO WORD GRAPHING

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S Graphs and K Graphs

Fig 1a is an S-graph (S for simple). It contains the number names FOUR, FIVE, SIX, SEVEN, EIGHT, NINE through NINETEEN, TWENTY, THIRTY, FORTY, FIFTY, SIXTY, SEVENTY, EIGHTY and NINETY. The letter Y joins the letters F, S, and E, thus allowing the compounds TWENTY-FOUR to NINETY-EIGHT -- 40 compounds, 64 names in all.

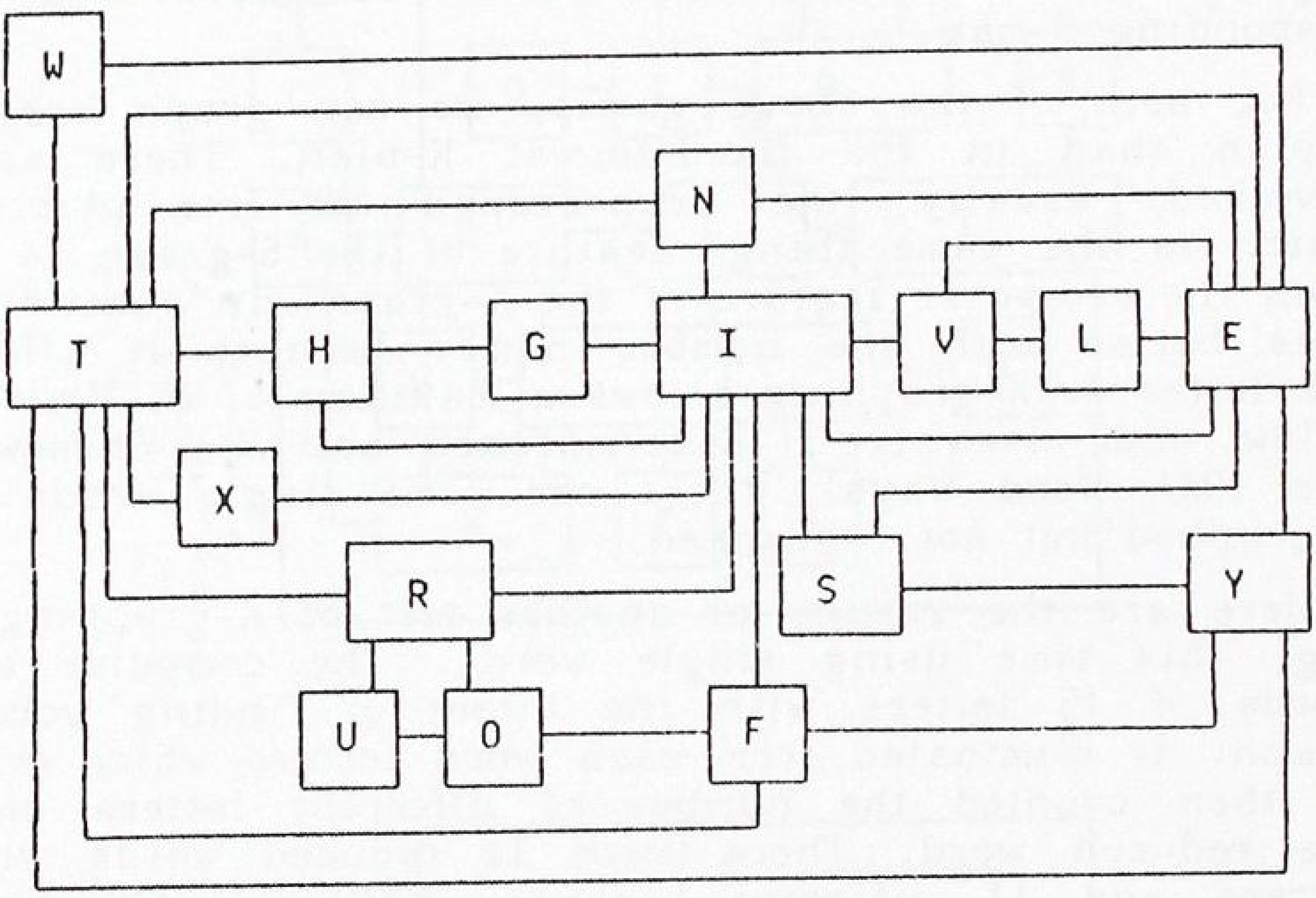


Fig 1a

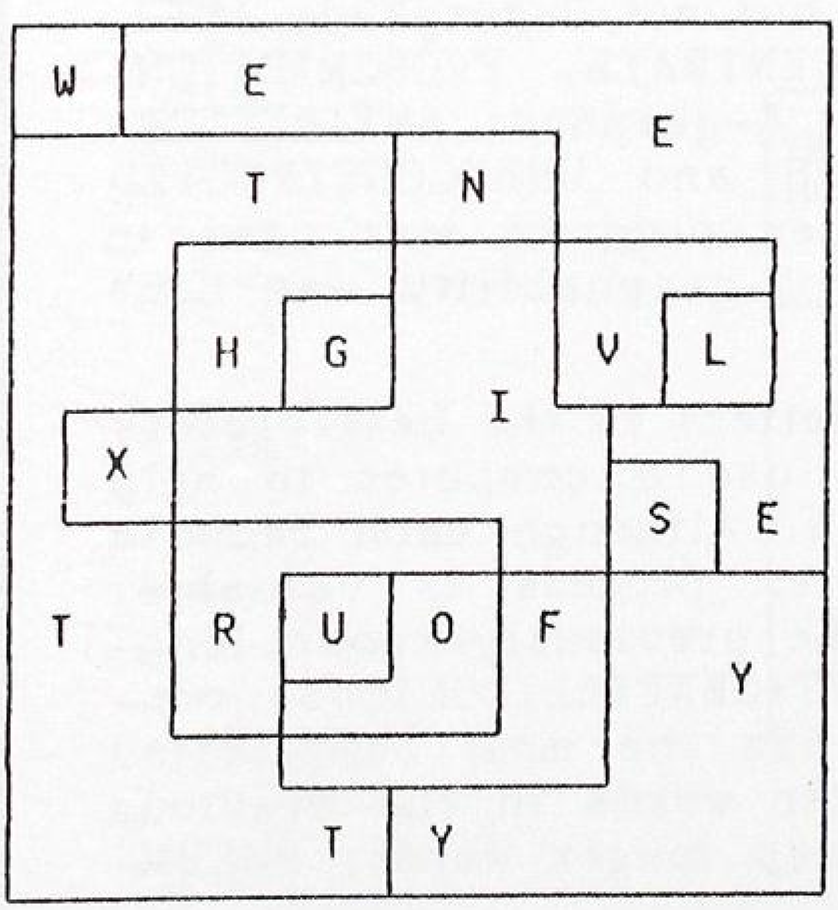


Figure 1b, the reciprocal of 1a, presents the same information. Fig 1b is also a graph, but let's call it a map to distinguish it from a line graph. In this S-map, regions touch only if a corresponding join exists on the line graph. Diagonal crossing is not allowed.

Fig 1b

Fig 2 is a K-graph (K for king's move, in chess). It contains the number names TWO, FOUR, SIX, SEVEN, NINE, TEN, ELEVEN, TWELVE, FOURTEEN, SIXTEEN, SEVENTEEN, NINETEEN, TWENTY, FORTY, SIXTY, SEVENTY, NINETY. The letter Y joins F, S, and T, thus allowing 20 compounds from TWENTY-TWO to NINETY-NINE - 37 names in all. This is actually a map. Diagonal crossing is allowed; points where this occurs are marked with an x. As used here, the term simple means that lines joining nodes do not cross. A K-graph is not simple, but it is still planar. It also has a fixed format.

Figs 3, 4, and 5a all contain VIOLET, BLUE, GREEN, YELLOW, ORANGE and RED. GREY comes in automatically in all cases. INDIGO doesn't fit anywhere. One of the K-graphs adds PINK, the other adds WHITE. Each of the K-graphs uses one crossover to allow the added color. K-graphing does not allow adding more than one color name from the set BLACK, WHITE, or PINK. The S-graph, Fig 5a, allows adding two: BLACK and WHITE. Fig 5b is the corresponding S-map.

For both of the above games, we can graph more words in the S-plan than in the fixed-format K-plan. There are two factors involved. Usually, the wrap-around (or free placement of nodes which is the same thing) feature of the S-graph is more valuable than the crossover feature of the K-graph. In addition, the S-graph does better with the number names because it allows 9 joins to the letter I. K-graphing allows a maximum of 8. However, as shown below and elsewhere ("Eodermdromes and Non-Chesswords", November 1981 **Word Ways**), there are some single words which can be K-graphed but not S-graphed.

Here are the results of another test of K-graphing vs. S-graphing, this time using single words. The computer examined 7,800 words of 15 letters with the intent of finding words difficult to graph. It eliminated from each word letters which occur only once. It then counted the number of different letters and bigrams in the reduced word. There were 11 reduced words with 6 different letters and 11 different bigrams. HYPERPHARYNGEAL cannot be S or K-graphed; ARTERIOPLASTIES can be K but not S-graphed; AURICULOCRANIAL, DIPLOSPONDYLISM, OVERCONCENTRATE, PROSCRIPTIONIST and UNCONSCIENTIOUS can be S but not K-graphed; ANTIRESTORATION, CRYPTOPROSELYTE, PERIOPTOMETRIES and UNRECONSTRUCTED can be graphed in both plans. A computer program was used to test words for K-graphability. Testing for S-graphability was done with pencil and paper.

Stripping words of singularly-occurring letters is the best process I have been able to devise in order to use a computer to help find words difficult or impossible to graph. Although each reduced word must be further tested, the computer process is valuable. For example, it quickly determined that the previously-known longest S-graphable word, SUPERCALIFRAGILISTICEXPIALIDOCIOUS, contains a lot of insignificant letters. What is the most complicated word which can be S-graphed? The 15-letter words in the previous paragraph make a start. Figs 6a and 6b map longer words: PNEUM-

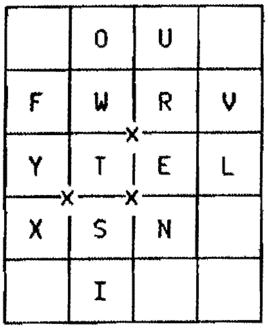


fig 2.

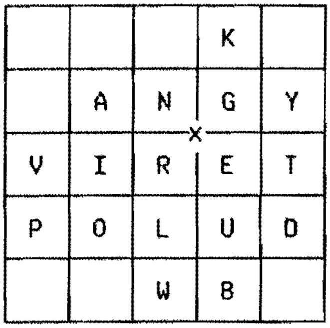


fig 3.

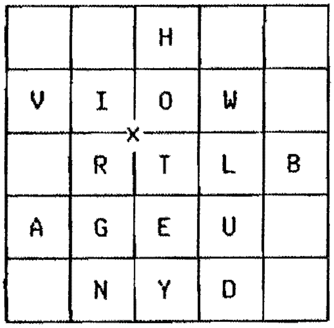


fig 4

Fig 5a

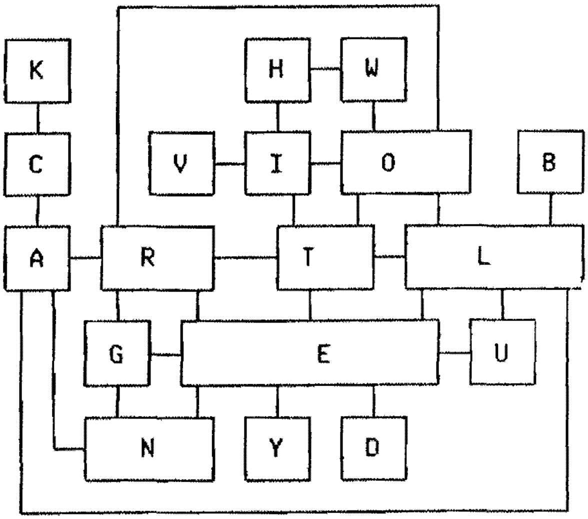
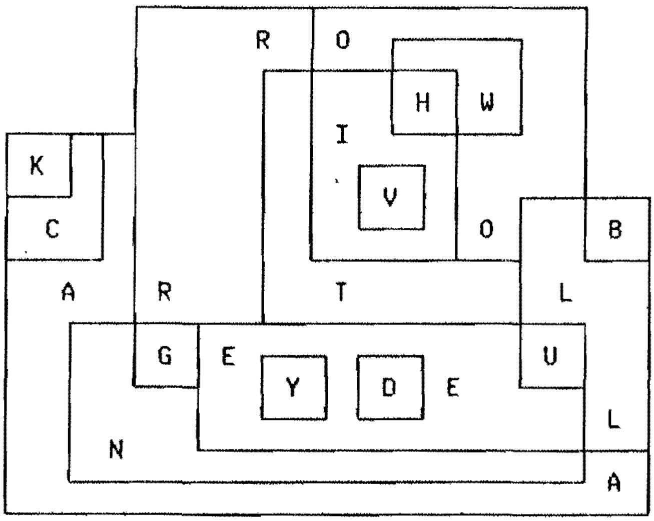


Fig 5b



ORETROPERITONEUM and SEMIMICRODETERMINATION. The reduced form of the first has 8 letters and 13 bigrams; the second has 7 letters and 13 bigrams. The prize goes to DACRYOCYSTORHINOSTEN-

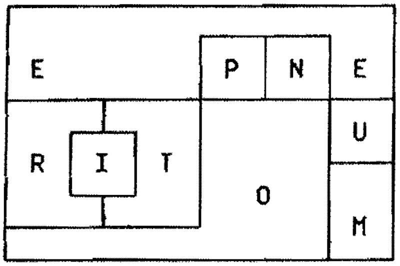


Fig 6a

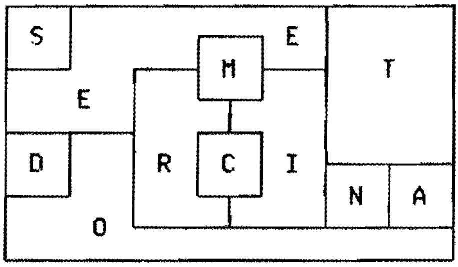
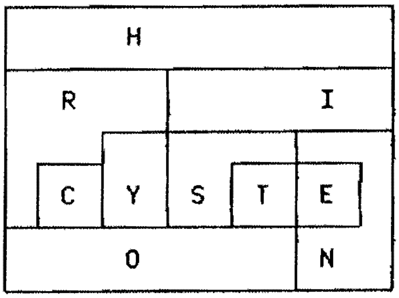


Fig 6b

Fig 6c



OSIS, mapped in Fig 6c. The reduced form has 8 letters and 15 bigrams. The booby prize goes to VENTRICULOCISTERNOSTOMIES; its reduced form has 8 letters and 17 bigrams. It is almost S-graphable. Although I have not investigated the subject in detail, it appears that all words for which bigrams exceed letters by less than 5 are S-graphable. Words in which the excess is greater than 6 are rare.

In a previous paragraph, I mentioned that using wrap-around joins in S-graphing is equivalent to allowing free placement of nodes with straight-line joins. Fig 7a and Fig 7b illustrate this point with alternate graphs of PSEUDOINTERNATIONALISTIC. Fig 8a presents the alternate line graph of DACRYOCYSTORHINOSTENOSIS, and Fig 8b shows it as a word molecule. The equivalence between S-graphs and word molecules is discussed in the August 1994 Kickshaws.

Fig 7a

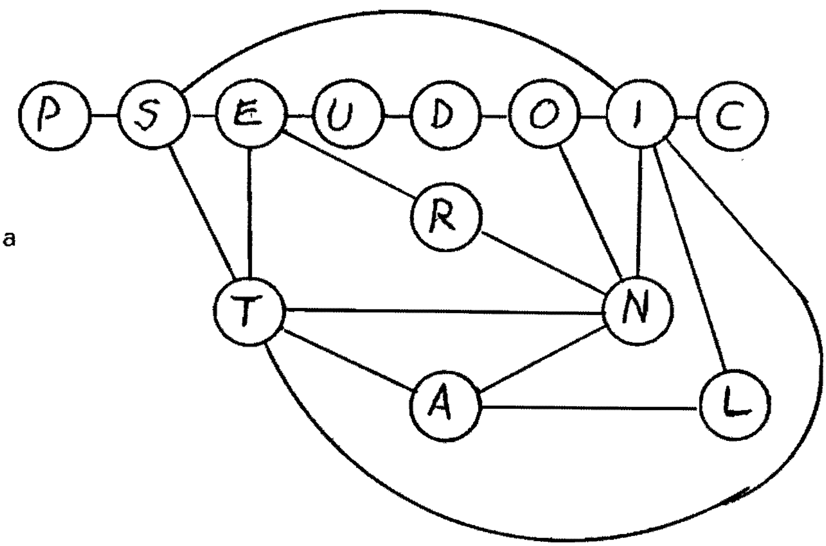
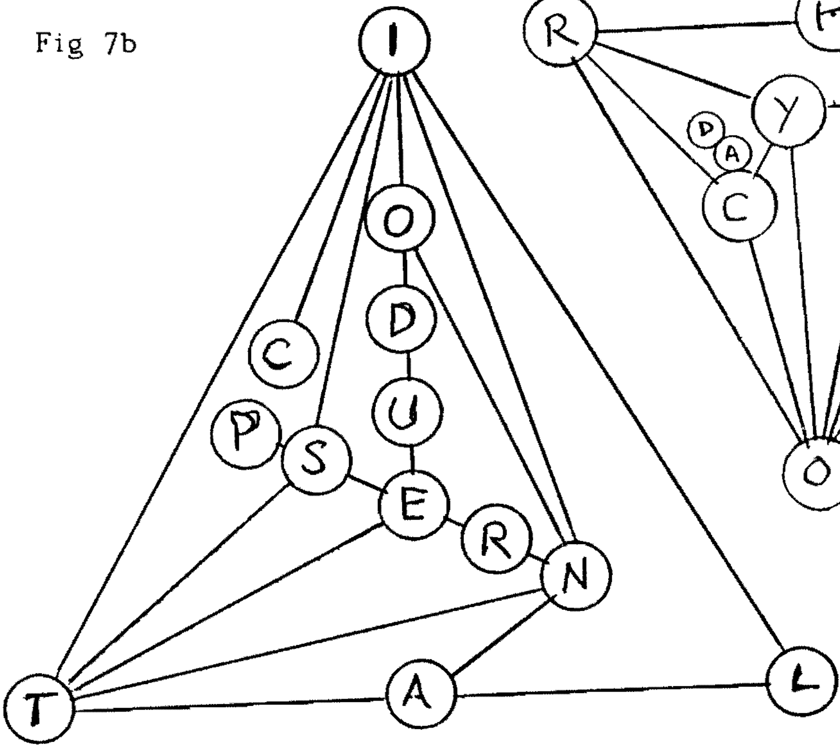


Fig 7b



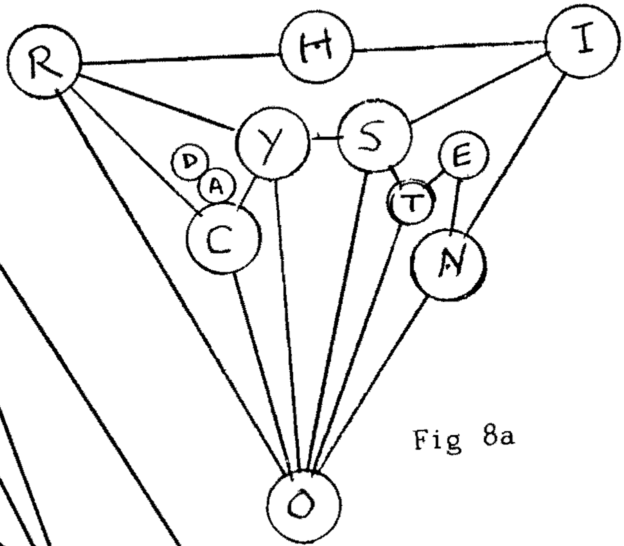
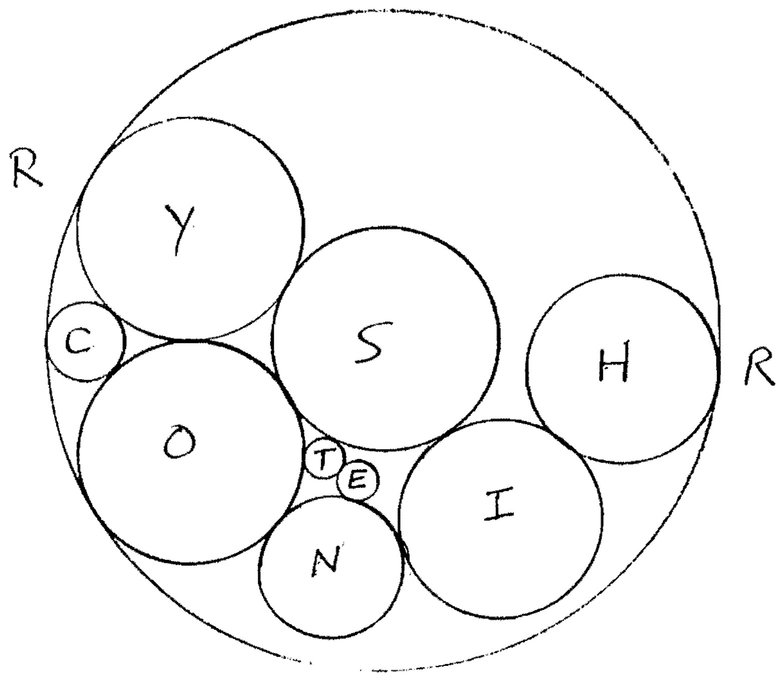


Fig 8a

Fig 8b



To complete this portion of the study, let's see what the longest *K*-graphable word is. There was no preliminary screening; I merely computer tested all the words in my fairly short list of words with 27 or more letters. Three are interesting: ETHYLENEDIAMINE-TETRAACETATE, OCTAMETHYLPYROPHOSPHORAMIDE, and DIAMINOPROPYLTETRAMETHYLENEDIAMINE. They are mapped in Fig 9. The first two can also be S-graphed; the third can not.

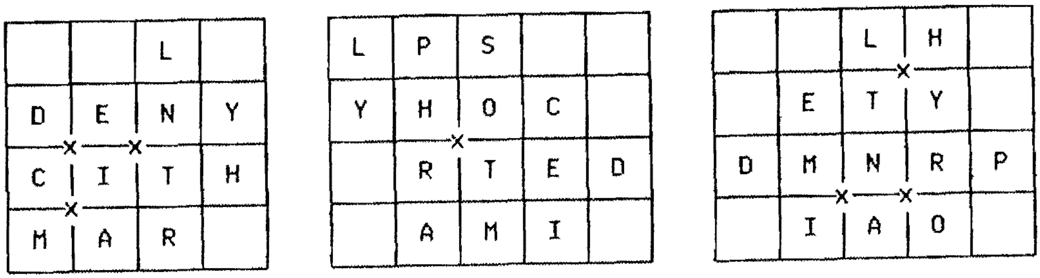


Fig 9

6J Graphs

Fig 10 and Fig 11 introduce two new graphing plans. Both are shown as line graphs and as maps. Let's call these OS-graphs (OS for octagon-square tiling of the plane) and H-graphs (H for hexagon tiling). Both are simple; i.e., lines between nodes do not cross. However, these plans do not allow the free placement of nodes enjoyed by the S-graph. Fig 10 and Fig 11 show the best I was able to do with the number-names game described at the beginning of this article.

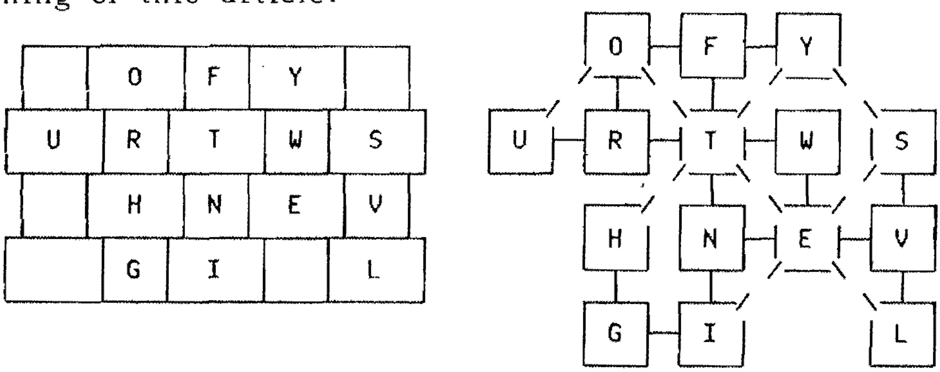


Fig 10

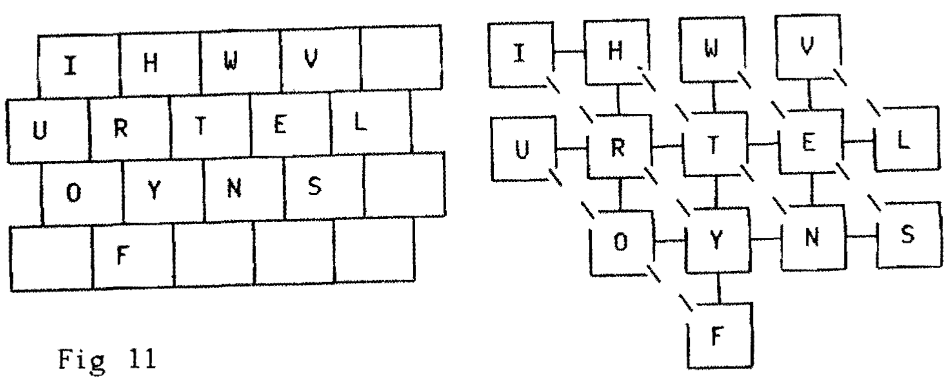


Fig 11

The August 1994 **Word Ways** contains an article in which I describe a maze; allowable paths are ana-gram-mar chains. This is not much different from finding paths in which letters spell words. A maze given in the Feb 1994 Games magazine used king's moves. My maze used the plan shown in Fig 12. Let's call this a G-graph (for Gordon). This is not a simple graph, but actually a restricted K-graph; a king can move only NW-SE on one color of the chessboard and NE-SW on the other. (The H-graph and the OS-graph also represent restricted king's moves.)

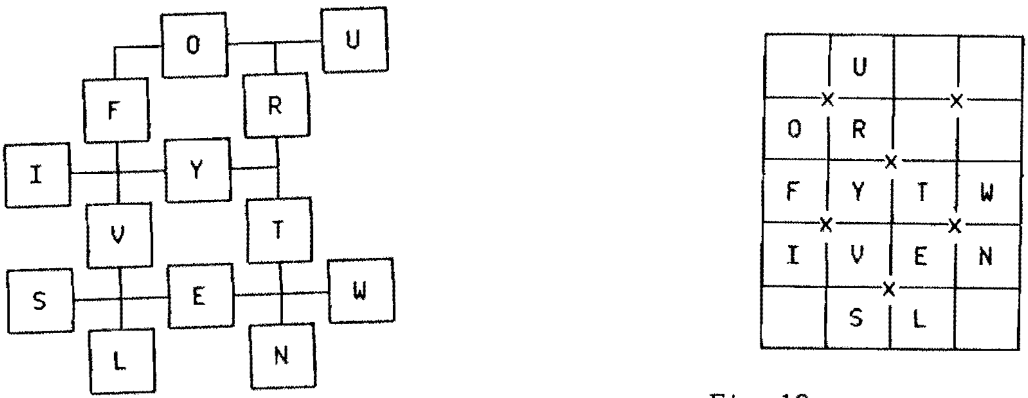


Fig 12

The G-graph, H-graph and OS-graph all have an average of 6 joinable positions: let's collectively call these 6J-graphs (the K-graph is 8J). It is clear from the definitions of the graphing plans that (1) any words which are H, G or OS-graphable must also be K-graphable, (2) any words which are H or OS-graphable must also be S-graphable. The G-graph is not simple, so there is no certainty that all G-graphable words are also S-graphable, but I have not found any words which were not. So, apparently all the words which can be graphed in any of the 6J plans discussed so far can also be K and S-graphed. The reverse is not true. As shown below, the OS plan does the best among the 6Js. None of the words of 15 letters listed earlier are OS-graphable. The following paragraphs compare 6J graphs with one another.

Requiring that words of 11 letters have 5 different letters, each used at least twice, my computer turned up 6 words. All can be S-graphed. Here is how they fare in the other plans: DENIZENIZED, GONOGENESES, and PARAPARESES can be graphed in all 4 plans; HORSESHOERS can only be OS or K-graphed; TRINITARIAN can only be K-graphed; INTENSITIES can not be graphed in any except the S-plan.

The following 13 words of 13 letters can be K and S-graphed. SEMIBARBARISM can be graphed in all 6J plans; CONSTRICTIONS, CORTICOTROPIC and OSTEOSTEATOMA can be OS-graphed; ARTHRESTHESIA, MESODUODENUMS, PHYSIOPSYCHIC, REINTEGRATING, RESTAURATEURS, SEMISTAMINATE, SUPERSATURATE and UNDERCHURCHED cannot be graphed in any of the 6J schemes.

Now, let's see how the three 6J plans compare when graphing words of 20 letters. I had the computer test all 360 words of this length in my database. 66 were H-graphable, 59 were G-graphable and 130 were OS-graphable. Surprisingly, all H-graphable words are also OS-graphable. Most of the G-graphable words are also

OS-graphable. There is no obvious geometrical reason for this. It is probable that the OS plan better fits the nature of English words, but it is hard to say why. The OS-graph allows 8 joins to some letters, but few individual words need it. The only point that is apparent is that H-graphs tend to be tighter than the others, while the OS-graphs tend to spread out so more letters can be placed on the octagons.

The first line of Fig 13 presents the longest H-graphable words, of 23 letters: MAGNETOHYDRODYNAMICALLY, PSEUDOPHILANTHROPICALLY and XANTHOERYTHRODERMICALLY. The second line shows the same words as OS-graphs. Other 23-letter OS-graphable words are not H-graphable.

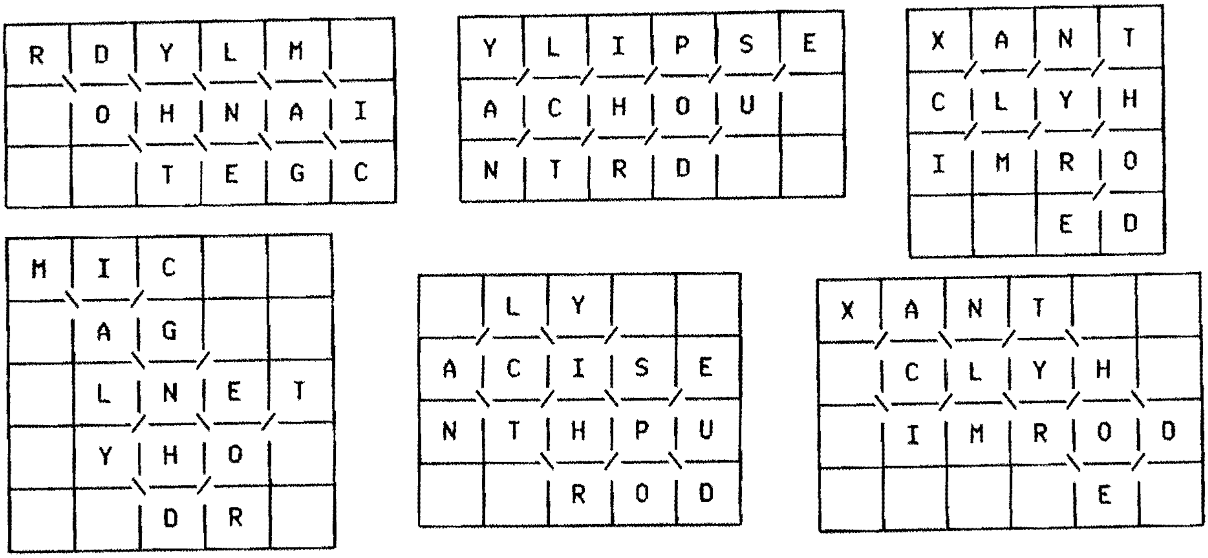


Fig 13

The longest G-graphable word is the 23-letter DICHLORODIFLUORO-METHANE. The next two maps show this word as an OS-graph and a K-graph. The final map is for the longest OS-graphable word, the 24-letter DIOXYDIAMIDOARSENOBENZOL. All four maps are shown as Fig 14 on the facing page.

Square and Triangle Graphs

Making the rook's move in chess a basis for graphing presents an interesting challenge in that a rook can move more than one space at a time. At this point, I separate the problem into two phases. Let's first just allow single steps to get another reduced form of the K-graph, and examine the rook move later. I call this first phase a Q-graph (Q for square). This simple graph has only 4 joins per node and it requires that repeated letters be an even distance apart. Any Q-graphable word is H-graphable as well. It is a nice project to search for Q-graphable long words. Graphing words with only one or two repeated letters is trivial; beyond that, the game gets interesting. Fig 15 shows 7 words of length 18 that are Q-graphable (BATHYHYPERESTHESIA, HYPERCO-

AGULABILITY, LARYNGOPHARYNGITIS, OVERSUSCEPTIBILITY, SUPERGRATIFICATION, SUPERMAGNIFICANTLY and SYNCATEGOREMATICAL) and 2 words of length 19 (HYPERTHYROIDIZATION, HYDROCINNAMALDEHYDE).

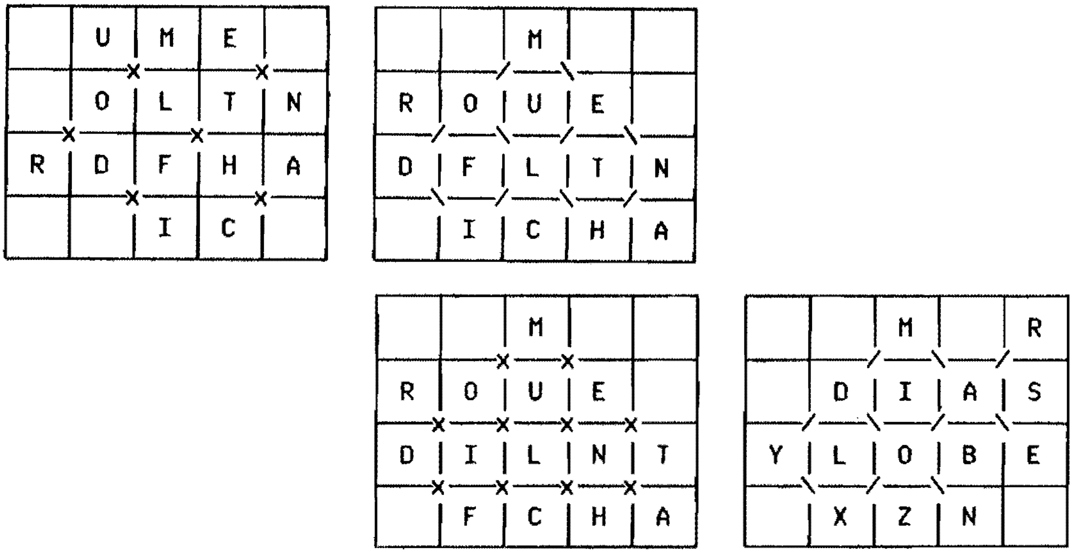


Fig 14

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B          H Y P E R   L             Y         E R G
A T H Y      T I B C   A R Y N     O P I T B   P A T
I S E P        L A O   H P O G     V E R L     U C I F
    R          U G         T I S     C S U     S   O N

  C I F      L              D        C I N
P E N T    N C A M        Z I O N    O R A M
U R G L    Y I T E G      A T R E    E D L
S M A Y    S     R O      H Y P      H Y
```

Fig 15

Fig 16a shows a plan for T-graphing (T for triangle).Fig 16b is the corresponding line graph. This plan allows face-to-face joins only. It is obviously a reduced Q-graph. Excluding words which do not produce any loops in the graph, the longest T-graphable words are the fifteen-letter OVERSTIMULATIVE, PITHECANTHROPID, SINISTRODEXTRAL and STIBIOCOLUMBITE, the sixteen-letter ARACHNODACTYLIES and TELEPHONOGRAPHIC, and the seventeen-letter DICYCLOPENTADIENE. The last four are mapped in Fig 17.

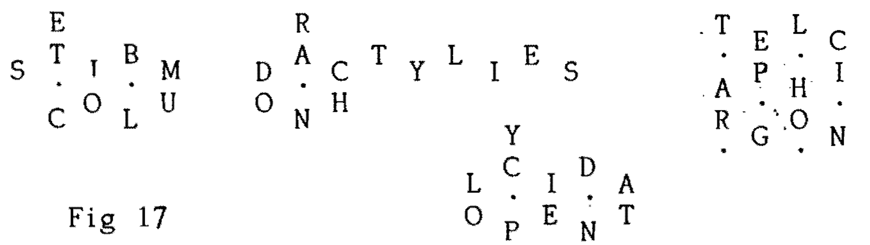

Fig 17

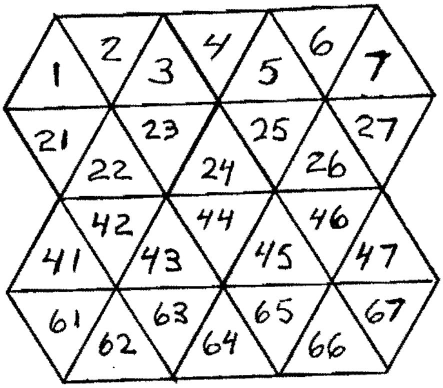

Fig 16a

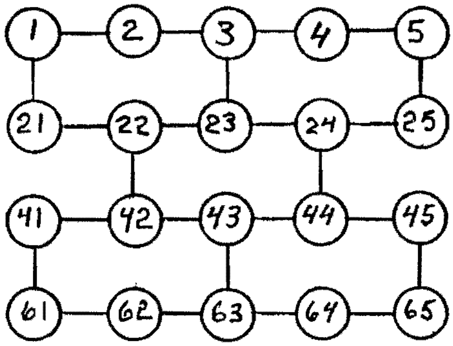

Fig 16b

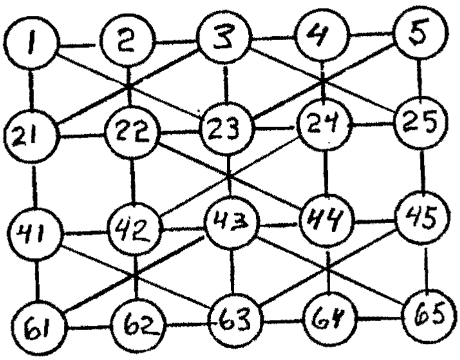

Fig 18

The XT-graph (shown in Fig 18, at the left) is something new. It is 6J, but is not a simple graph. Instead, it is a Q-graph with added crossing joins. The added moves correspond to chess knight moves being allowed in certain places. The strange thing here is that just like the Q-graph, repeated letters have to be an even distance apart. The longest XT-graphable words are the twenty-letter DIMETHYLTUBOCURARINE, OPHTHALMODIASTIMETER, and TYCHOPARTHENOGENESIS. All three are mapped in Fig. 19. The first and third words can be OS-graphed and K-graphed, but the second can be neither OS-graphed nor S-graphed. Its K-graph contains four places where joins cross; none of the K-graphs mentioned in the first part of this article use more than three.

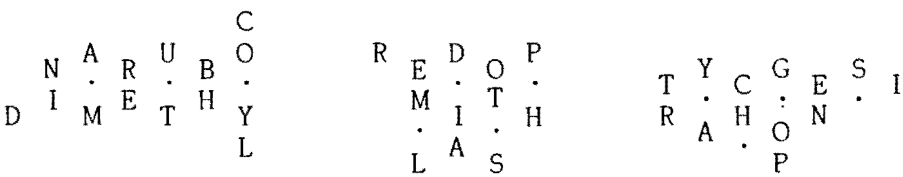

Fig 19

I now return to studying R-graphs (R for rook's move in chess). I have not developed a satisfactory computer program for this plan which allows the distance between joined letters to be as long as needed so that there are no intervening letters. However, I was lucky in that for the longest words, preliminary screening ruled out all but four 22-letter ones which I was able to test with pencil and paper. LARYNGOPHARYNGECTOMIES, VENTRICULOSUBARACHNOID and SULFAMETHOXYPYRIDAZINE are R-graphable, as shown in Fig 20. This plan does not do as well as the several 6Js which allow words of 23 or 24 letters, but it is better than the Q-graph which only allows words of 19 letters. Note that in the first R-graph the **E-I** and O-**T** joins cross. All three words can be K-graphed; the K-graph of the second has five places where joins cross.

L					S	
A	R	Y	N	G	E	C
H		P		O		T
				M	I	

			D			
V	O		I		C	
E	N	T	R			
	H	C	A	B	U	L
					S	O

U	L	M		E	T	H	
S	F	A	D			O	
		Z	I	N		X	
			R			Y	P

Fig 20

Three-D Graphs

Although I am not keen about 3-D puzzles and games (one can usually get as much variety as one needs from 2-D plans), let's do a little bit of 3-D work here for comparison. The following is a C-graph (C for cubic) of a 20-letter word: DIMETHYLTUBOCURA-RINE. Read Fig 21 as if the second set of letters is directly above the first. This is the longest word graphable in this plan.

	R	A	
C	U	T	H
O	B	L	Y

D	I	M	
	N	E	

Fig 21

My second 3-D plan consists of a series of hexagonal fields directly above one another. This 8J P-graph (P for prism) does better than the 3-D C-graph, but not as well as the planar 8J K-graph. Fig 22 maps five words of 23 letters or more as P-graphs: EPIDIDYMODEFERENTECTOMY, BLEPHAROSPHINCTERECTOMY, DICHLORO-DIFLUOROMETHANE, DIOXYDIAMIDOARSENOBENZOL, OCCIPITOROSCIPITO-SCAPULAR. The fifth, of 25 letters, is longer than any 6J word I've found but is short of the longest 8J K-graphable words. In Fig 22d, there is an L on a third layer, below the O.

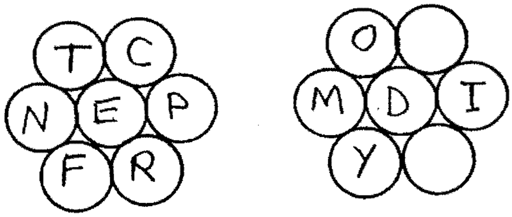

Fig 22a

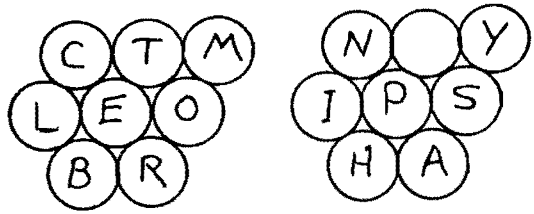

Fig 22b

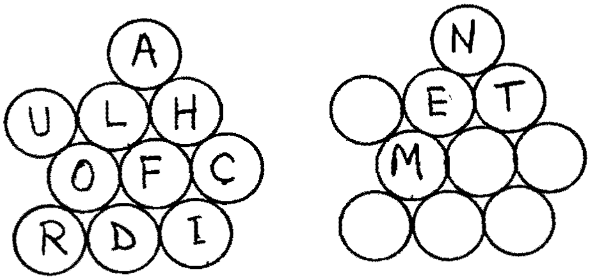

Fig 22c

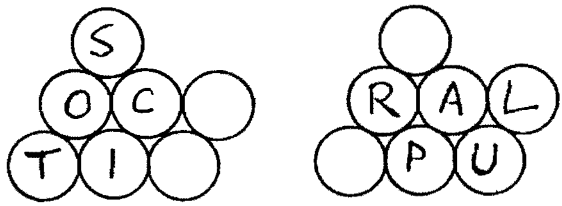

Fig 22e

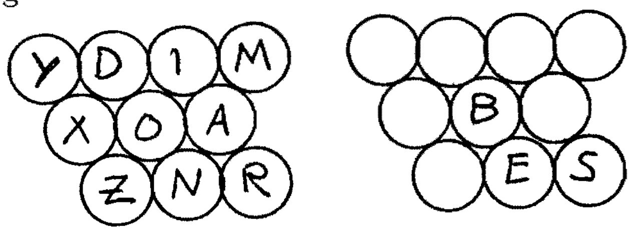

Fig 22d

Future Work

Throughout the above, I have compared graphing plans mainly by finding the longest graphable words in each plan and by giving examples of words graphable in this and not that plan. However, I have not addressed the problem of what is the shortest word graphable in this and not that plan. This was done adequately for S-graphs vs. K-graphs in earlier **Word Ways** articles, but now we have a lot more plans to compare.

Can a 3-D close packing plan (a fixed format 12J one) do better than K-graphs or S-graphs?