

NON-TRANSITIVE WORD GAMES

JEREMIAH FARRELL

Indianapolis, Indiana

DAVID WRIGHT

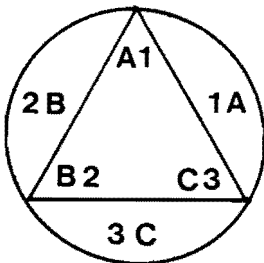
Boston, Massachusetts

"Longer than" is a binary relation R on the set of words with the property that whenever $a R b$ and $b R c$, we always have $a R c$. Such relations are called transitive and examples abound: "shorter than", "precedes lexicographically", "scores higher in Scrabble than", etc. Transitive relations often impose a kind of "can you top this?" attitude among logologists: "what is the longest word you know?" In this article we wish to avoid transitive relations, and instead play word games that have no "best" answer.

The familiar Stone-Paper-Scissors game is an example. As described by Konrad Jacobs in *Invitation to Mathematics* (Princeton University Press, 1992), we have "paper wraps stone", "stone blunts scissors" and "scissors cuts paper". It is possible to play a game of Stone-Paper-Scissors with the reader by long distance using the following grid.

3	PAPER	SCISSORS	STONE
2	STONE	PAPER	SCISSORS
1	SCISSORS	STONE	PAPER
	A	B	C

You (the first player) choose one of the rows 1,2 or 3 or else one of the columns A, B or C; we (the second player) choose one of the remaining two rows or columns. You then choose one of the three intersecting choices to decide the winner. For example, if you choose C and we choose B, you can choose 3 and win by stone (C3) blunts scissors (B3). (If you choose 2 or 1 instead, you will also win.)



To play this game, we must have a way of making our selection; we employ the magic amulet for this purpose. After you have made your choice (1,2,3 or A,B,C) put your finger on that letter or number in the circle outside the triangle. Choose a word (stone, paper or scissors) and spell it out on the points of the inner triangle (either clockwise or counterclockwise). The point you land on will be our choice--a letter if you chose a letter, a number if you chose a number.

The second player always wins at this game. This is typical of non-transitive games, although the edge is not usually as high as 100%

This type of nontransitive word set can be extended. For instance, using the four classical elements of the ancient Greeks we can obtain FIRE scorches EARTH displaces AIR evaporates WATER quenches FIRE. (To complete the comparisons, add FIRE feeds on AIR and EARTH muddies WATER.) The grid below illustrates a placement of the elements which creates a game that works like Stone-Paper-Scissors.

EARTH	AIR	WATER	FIRE
FIRE	EARTH	AIR	WATER
WATER	FIRE	EARTH	AIR
AIR	WATER	FIRE	EARTH

The second non-transitive word game, Chow, is played using the four-by-four word square given below. If we choose at random corresponding letters from two successive words, we see that CHOW, for example, precedes LOVE with a probability of $3/4$ (C precedes L, H precedes O, O precedes V, but W does not precede W in the alphabet). Similarly, LOVE precedes OPEN, OPEN precedes YENS, and YENS precedes CHOW, each with probability $3/4$. A similar order of precedence holds on the words of the columns. Two persons play Chow by each selecting one word from the rows (or from the columns) and spinning a spinner or rolling a die to decide which letter is to be chosen from their words; the player with the earlier alphabetic letter is the winner.

C	H	O	W
L	O	V	E
O	P	E	N
Y	E	N	S

This game is completely fair when there are four players (each has a probability $1/4$ of winning), and gives a winning probability of $3/4$ to the second player (who chooses the word above the first player's) when two play. When there are three players, the results depend on whether the second player knows the game strategy. If the first two players merely pick words at random, they will pick adjacent words $2/3$ of the time (CHOW and YENS are considered to be adjacent) and non-adjacent words $1/3$ of the time. The third player has a winning probability of $1/2$ in the former case (by picking either remaining word) but only $1/4$ in the latter case, for an average probability of winning of $5/12$. However, if the second player knows the strategy, he will pick a non-adjacent word; the third player then has a probability of $1/4$ of winning, and the first two players each have a probability of $3/8$ of winning. It pays to go last in the two-person version of Chow, but not the three-person one.

Word Bingo is a satisfying two-player game that can be played over and over without the players ever becoming aware there is a nontransitive advantage for the second player. The ten possible two-letter combinations of BINGO are placed as indicated below on five Bingo cards. Players choose a card to play, and letters are drawn without replacement from the letters of BINGO, to be used by all, with the winner being the first to form a letter-pair on his card. If there is a tie, the BINGO letters are mixed again and replayed.

BI	GI	NG	GB	OB
NB	IN	ON	GO	IO

To gain an edge in Bingo, the second player to draw a card notes that on each card one letter appears twice: the name of the card. Card B dominates Card I dominates Card N dominates Card G dominates Card O. Thus, the second player should draw a card to the left of the one drawn by the first player. Of the 120 orders in which BINGO can be drawn, 60 will lead to wins by the second player, 50 to wins by the first player, and 10 to ties: the first player has a 6:5 edge. If the second player selects a card two places to the left (for example B, if the first player selects N), then the second player's edge is reduced to 28:25. The second player should sometimes select this less-productive strategy in order to keep the first player in the dark concerning his strategy.

It is impossible to make the game logologically attractive by replacing the ten letter-pairs with ten two-letter words. However, if the letters of DENIM are used, the two-letter pairs are all state abbreviations except for IE. A three-letter version of Bingo can be created out of YEAST, with all words but TAY common ones. For yet another Bingo word game, we suggest using the letters of MAUSER, since every subset of three letters can be transposed into a reasonable dictionary entry.

Our last game, Salt Mine, is more of a hoax than a true nontransitive set, but since it gives the second player a 3:1 edge we offer it here. Obtain four wooden tokens and stamp the letters S, A, L and T on their faces and M, I, N and E on their respective reversals. A different word can then be formed whenever the tokens are tossed. One set of words is given at the left. Each word is from the OED except SINT which is a Norwegian word meaning cross or angry. Each word abuts four others, including the top row which joins the bottom and the right column which joins the left (forming a torus, or doughnut).

MANE	SANE	SINE	MINE
MALE	SALE	SILE	MILE
MALT	SALT	SILT	MILT
MANT	SANT	SINT	MINT

The two players begin a game by each choosing one of the four cards illustrated below. For clarity, the words have been transposed from the previous sixteen into more common ones. The tokens are tossed, and then turned one by one until someone is able to form a word on his card and wins.

To gain the edge, you must be the first person to turn a token, since, amazingly, any one of the 12 words not on a given card is only one token turn away from some word on it. This is far from obvious when you are studying the cards but becomes transparent when looking at the toroidal

graph.

To assure that we turn first, we make the offer to our opponent "Do you want to toss first or second?" If he wants to be first, we hand him all four tokens to be tossed at once. If he wants us to go first, we select one token, toss it, and tell him it is his turn to toss the next token. In either case, tossing all at once or one-at-a-time, if our opponent does not win immediately after four have been tossed (he has a probability of 1/4 of doing so), it will be our turn to turn a token, and we will always be able to form a word on our card and win!

Bright Eyed and Bushy Tailed

Speaking of Animals: A Dictionary of Animal Metaphors (Greenwood Press, 1995) is a collection of approximately three thousand words or phrases, from A to ZOO PLANE, related in some way to animals, birds, fish and insects. The author, Robert Palmatier, includes not only true metaphors like eat like a horse, blind as a bat or crazy as a loon, but more general constructions like foxhole or litterbug. For some, such as ukulele, stand the gaff or egg on your face, short explanations are required. (A ukulele is a jumping flea, a fighting gamecock wears a metal spur called a gaff, and a weasel with egg smeared on its face is judged guilty by the farmer.) If one is interested in a metaphor for a specified animal, the author provides an index by name at the end of the book. This book is both comprehensive and entertaining; nevertheless, the price of \$69.50 may make all but the dedicated metaphorist think twice.

Note added in proof: LEMON and MELON are not reversals in the Palindromic Slide Rule on the back cover; substitute PAR and RAP, or LEER and REEL.