SQUARE AND TRIANGULAR NUMBER-NAMES

MONTE J. ZERGER
Alamosa, Colorado

Bi-Squares

   In the mathematics of sets the cardinality of a collection of objects
is defined to be the number of objects in that collection, and is
denoted by placing vertical bars around a listing of that collection.
Following that convention we write ¦FOUR¦ = 4 or ¦SEVEN¦ = 5.

   Let's call a number *bi-square* if it is a square number and the
cardinality of its name is also a square (allowing the letters in its
name to be arranged in a square pattern as illustrated below). ¦FOUR¦ =
¦NINE¦ = 4 and ¦THIRTY-SIX¦ = ¦FORTY-NINE¦ = ¦SIXTY-FOUR¦ = ¦EIGHTY-ONE¦
= 9 are the only bi-squares less than 100.

```
F O     N I     T H I     F O R     S I X     E I G
U R     N E     R T Y     T Y N     T Y F     H T Y
                S I X     I N E     O U R     O N E
```

Though FOUR probably deserves the bi-square prize (it correctly
proclaims the number of letters in its name as well), NINE surely comes
in a close second since NINE, its square and its cube are bi-squares.

```
N I     E I G     S E V E N
N E     H T Y     H U N D R
        O N E     E D A N D
                  T W E N T
                  Y N I N E
```

   The letters in the names of the first nine numbers can also be
arranged in the shape of a square. The cardinalities of these number
names neatly divide into three congruent classes: ¦ONE¦ = ¦TWO¦ = ¦SIX¦
= 3, ¦FOUR¦ = ¦FIVE¦ = ¦NINE¦ = 4, ¦THREE¦ = ¦SEVEN¦ = ¦EIGHT¦ = 5.
Consequently they can be arranged to form a magic square of
cardinalities. The total number of letters in each of the three rows,
three columns and two diagonals is the same.

```
O N E T W O          ¦FOUR¦     ¦THREE¦     ¦ONE¦
T H R E E F          ¦TWO¦      ¦FIVE¦      ¦SEVEN¦
O U R F I V          ¦EIGHT¦    ¦SIX¦       ¦NINE¦
E S I X S E
V E N E I G
H T N I N E
```

Bi-Triangulars

We can ask a similar question about triangular shapes. In mathematics a number is called triangular if it can be represented as the sum of successive numbers beginning with 1 (that is, 1, 3, 6, 10, 15, 21, ...). The objects in a collection whose cardinality is a triangular number can be arranged into the shape of an equilateral triangle. Bi-triangular numbers include SIX and TEN, and it is additionally pleasing that ¦TRIANGULAR¦ = 10, a triangular number.

```
    *       or   S       *       or   T              T
   * *              I X      * *              E N        R I
  * * *                     * * *                       A N G
                           * * * *                     U L A R
```

The Triangular-Square 36

36 is a triangular and a square number, making it possible for 36 objects to be arranged into either a square or triangular pattern. Given the rarity of such numbers (there are only two others less than one million), it is rather amazing that the cardinalities of the names of the first nine numbers can be used to form either shape. The square shape has already been presented; the triangular one is given below.

Equally remarkable is the set formed by combining all ten digits and all 26 letters. With this set of characters we can form all numbers in mathematics, and all words (excluding special characters) in the English language. Since its cardinality is also 36, these characters can also be arranged into a square or a triangle, the triangle having the additional appeal of a triangle of digits sitting neatly atop the letters.

```
         O              0 1 2 3 4 5              O
        N E             6 7 8 9 A B             1 2
       T W O            C D E F G H             3 4 5
      T H R E           I J K L M N            6 7 8 9
     E F O U R          O P Q R S T           A B C D E
    F I V E S I         U V W X Y Z          F G H I J K
   X S E V E N E                            L M N O P Q R
  I G H T N I N E                          S T U V W X Y Z
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Generalizations

Clever readers can expand on these ideas by investigating words which *imply* a number. SEPTEMBER can be considered a bi-square since it is the ninth month and its name contains a square number of letters. Likewise MARS, the fourth planet from the sun, and JOHN, the fourth gospel, are appropriately named. And so we end where we began, with the one number that is perfectly named. Observing that JOHN, WORD and WAYS all have this cardinality, I am reminded of the beauty of John's opening passage "In the beginning was the Word and the Word was with God, and the Word was God."