

PLAYING JOTTO AGAINST A COMPUTER

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The November 1989 Word Ways reviewed Recreational Mathematical Software (Clarks Summit PA) designed for playing the game of Jotto with a computer. In the two-person game of Jotto, each person selects a five-letter target word; the object is to guess your opponent's word before he guesses yours. To identify his target word, you propose a sequence of five-letter guess words, and your opponent tells you how many letters of the guess word are in his target word. For example, if the target word is PIVOT, THOSE scores 2 (T,O match) and MUNCH, 0. You and your opponent agree beforehand on a dictionary of allowable guess and target words (the Official Scrabble Players Dictionary is useful here).

In this article, we impose restrictions on the vocabulary: all words must be isograms (no repeated letters). The ideas presented in this article can be equally well applied to an unrestricted vocabulary.

The optimum strategy for playing Jotto is not known. What strategy is employed by Recreational Mathematical Software? As far as can be ascertained from studying actual games, the computer picks the first guess word at random, and later guess words which overlap earlier ones according to the scores you give him. To illustrate, suppose you select the target QURSH, and the computer selects BLOWS as its first guess. This has one letter in common with QURSH (S), so the computer must select a second guess word which has one letter in common with BLOWS, for example MOURN (O). This has two letters in common with QURSH (U,R), so the computer selects BEGUN as its third guess word. This has one letter in common with BLOWS (O) and two with MOURN (U,N). There appears to be no strategy for selecting the overlapping letters; sometimes they remain the same from one guess word to the next, sometimes not. It seems probable that the computer runs through its vocabulary and selects the first word it encounters that is consistent with the restrictions.

If the computer has identified an anagram of your target word, it does not run through the possible other anagrams in a predictable cycle.

This strategy, relatively easy to implement with the computer, is certainly not the optimum one. A better strategy, which would require massive amounts of computation, is the following. At the time one must select the next guess word, consider the reduced set of words that are

possible solutions in view of the guess-word scores up to that point. For each word in the original dictionary, consider how much the set of possible solution words will be further reduced by the various potential scores. For example, if there are 152 possible solution words, suppose that there will be 50,40,30,20,10 or 2 words still in the running if that guess word scores 0,1,2,3,4 or 5. In this case the worst potential outcome is 0, for this reduces the possible solution words the least (to 50). Carry out this computation for all dictionary words, and use as the next guess that word for which the worst set is as small as possible. For example, if another word results in 40,40,40,20,8 or 4 words still in the running, then this would be preferred since the worst case, 40, is less than 50.

This strategy potentially leads to guess words which cannot possibly result in an immediate win--i.e., words for which the score cannot be 5. To reduce the work factor, one can carry out the above-outlined strategy only for guess words that are possible solutions--a steadily smaller set to process as the game progresses. To avoid a massive computation at the start of every game, one can use a standard strategy with all possible branches identified. For example, assume that the word TAPER results in a partition vector of 1210,3031,1485,211,40 or 3 reduced sets corresponding to scores of 0,1,2,3,4 or 5, and that no other word has a largest partition vector value less than 3031--then TAPER is the first guess vector in every game. In similar fashion, determine the six second-step guesses corresponding to the partitions achieved by TAPER, and the 36 third-step guesses corresponding to the second-step guesses and their partitions. Hopefully, by the time the fourth step is reached in an actual game, the words still in the running will be a small fraction of the initial vocabulary, and one can customize the rest of the strategy as indicated at the start of this paragraph.

Playing against this computer program has one advantage over playing against a human: the program does not "remember" earlier targets you selected and modify its strategy accordingly. Nor does it try to take advantage of the standard strategy of your guesses of its word. This suggests that it is possible to look for target words which on the average give the program the most trouble, and use these *ad nauseam*. Because the length of a game varies considerably, it would be an exceedingly protracted task to try each word in the vocabulary a sufficient number of times to statistically distinguish the "best" from the others.

However, one can speculate that the most troublesome target word will either be a word which is isolated in five-dimensional word-space (i.e., having few words that share four or more letters with it), or else a word which is in one of the most crowded regions (i.e., having many words that share four or more letters). Two good examples of these are QURSH and STARE, respectively, with the matches tabulated below. For 5-matches, all anagrams are given; for 4-matches, only one anagram is needed.

QURSH

ursh: Brush, Crush, nurDs, ushEr, shruG, hurLs, hOurs, hurTs, rushY
 qush: quAsh

STARE (aster,tares,tears,rates)

star: Brats, Carts, Darts, raFts, trash, astIr, Karst, sMart, tarNs, roAst,
 Parts, sUtra, Warts, strAY
 stae: Baste, Caste, Dates, Fates, Gates, Hates, staKe, staLe, Mates, aNtes,
 Paste, saUte, Vesta, Waste, taXes, Yeast, Zetas
 stre: Crest, Drest, Frets, tIres, treKs, terMs, terNs, stOre, streP, Verst,
 streW, tYres
 sare: Bares, Cares, Dares, Fares, Gears, Hears, arIse, asKer, Laser,
 reaMs, earNs, sPare, Ursae, aVers, Wears, Years, raZes
 tare: taBer, Crate, traDe, aFter, Grate, Heart, Irate, taKer, Later, Mater,
 aNtre, Orate, Prate, aVert, Water, teary

We played ten games apiece with the target words QURSH and STARE respectively, making sure that we lost the game each time. The game durations were 9,8,9,8,9,7,11,11,10 and 8 turns for QURSH and 7,10,13,8,9,9,15,9,14 and 7 for STARE. The average number of turns is 9.0 for QURSH and 10.1 for STARE, suggesting that multiple alternatives necessitate about one additional turn to deal with them. However, it would take a sample some five to ten times as large to establish the reality of this apparent difference with reasonable credibility. It took the computer 5.1 turns on the average to reach the vicinity of QURSH with a guess word scoring 4; however, the corresponding vicinity of STARE was reached in only 4.9 turns (and twice on the very first turn!).

If the computer has picked a sequence of n guess words, is the $(n+1)$ st word that it will select uniquely determined? The answer is usually but not always. Trying to guess QURSH, the computer twice started with FEIGN, FIVES and TUBES with the following results:

FEIGN 0, BACKS, ...
 FEIGN 0, BOUTS, ...
 FIVES 1, CHILD 1, BATHS 2, FLOAT 0, HUNKS 3, HUMPS, ...
 FIVES 1, CHILD 1, BATHS 2, FLOAT 0, HUNKS 3, HYMNS, ...
 TUBES 2, DUMPS 2, JULEP 1, MINES 1, SHRUG 4, SUGAR 3, HOURS, ...
 TUBES 2, DUMPS 2, JULEP 1, MINES 1, SHRUG 4, SUGAR 3. CRUSH, ...