ASCENDING MAGNITUDES FOR K-GRAPHS

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In the May 1994 Word Ways, Lee Sallows published a K-graph containing the number words ONE through NINE. He then set several challenges, one of which was to find the K-graph which had the most numbers total, regardless of consecutivity.

A K-graph (king's-move graph) is a template of letters arranged in a square grid. A word is contained in it if its letters can be reached in order by means of king's moves in chess. Duplicate letters are not permitted in the graph; double letters are obtained by allowing null moves.

While attempting to find a K-graph for the numbers ONE through TEN (one of Lee's other challenges), I found a graph with thirteen numbers which was published in the November 1994 Colloquy. The maximum number in this graph was NINETEEN. In the following issue, Lee demolished that record with a graph containing 37 number names, the greatest of which was NINETY-NINE.

Thinking it unlikely that I could improve on this with numbers below one hundred, I decided to investigate those higher. I quickly realized that both HUNDRED and THOUSAND were easily accommodated in the same graph. My first effort was a modest improvement with 47 numbers, with the largest number being EIGHTY THOUSAND EIGHT HUNDRED EIGHTEEN (graph on the left).

```
  Y G F          Y G F
  H T I        O H T I
O U E V        U N E V
S N D R        A S D R
A
```

After trying several completely different approaches (with no luck), I realized that my previous best could be greatly improved merely by switching the O and the U. This simple change increased the count to 95, just over double the previous total. Continuing experimentation finally gave a much better result, both numerically and esthetically (graph on the right above).

This graph has the names for 1,5,7,8,10,15,17,18,50,51,70,71,80 and 81. In the 100s, there are 100,107,108,117,118,170,171,180 and 181. The same set of numbers are also there for the 500s, 700s and 800s. There are 24 numbers in the 1000 to 2000 range (1000,1007,1008,1017,1018,1070,1071 and

1081, plus the same numbers for the 1700s and 1800s). Numbers congruent to those in the 1000s are there for 5000,7000,8000,10000,15000, 17000,18000,50000,51000,70000,71000,80000 and 81000. All told, there are 428 number names with EIGHTY-ONE THOUSAND EIGHT HUNDRED EIGHTY-ONE as the highest.

Next, I tried graphs containing HUNDRED THOUSAND. The additional numbers generated thereby could not make up for constraint of requiring the D to be adjacent to the T. My best effort had only 189 number names (graph on left). Note that the X contributes only to SIX.

```
Y W
T H O
D E U
R N S X
  A I
```

```
  R
  D E T
I N Y H
A S U O
```

The high number in this graph is NINE HUNDRED NINETY THOUSAND NINE HUNDRED NINETY-TWO. This high a number made me wonder if it was possible to get 999,999 in a single graph. Without the requirement of trying for the most numbers, it turned out to be rather easy (graph on right above).

Unfortunately, including MILLION and BILLION in a template did not produce greater counts. The best I could do was 185 numbers (see graph below), the largest of which is SEVENTY-NINE BILLION ELEVEN MILLION ELEVEN.

```
    S
  M E V
Y N I L
T B O F
  W R U
```

I think it unlikely that including even higher numbers will increase the count. While I've hardly exhausted all possibilities, what I have done indicates that the highest counts are probably for those with a maximum number in the tens of thousands range.

I'll leave it as a challenge to the readers to find the K-graph containing the highest number.