WORD ARITHMETIC

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This article attempts to provide a taxonomy and nomenclature for that type of wordplay which involves manipulating the numerical equivalents of letters in words. Later articles will present results for two particular sub-species of manipulation: absolute difference words and cyclic difference words.

The earliest reference I have to difference words is in Dmitri Borgmann's *Beyond Language* (1967), although their invention has been ascribed to Howard Bergerson. A simple example is SOFA becoming DIE, where the positions of D, I and E in the alphabet (4,9 and 5) are the distances between the letters of SOFA.

Translation

The general principle involved is to *translate* the letters of a word into their numerical equivalents, and then to manipulate the numbers arithmetically before re-translating the resultant numbers back into letters to form a new word. Let us say that a source word is translated to a source *series of numbers*, which is manipulated into a *target series of numbers*, which are translated in reverse back to a *target word*. The translation may be done in many ways, of which the most common is A=1, B=2 etc, hyphens and apostrophes usually being valued at null, and upper case letters the same as lower case. When a zero is produced by arithmetic manipulation, it is commonly treated as invalid, so that source words with successive letters the same will not produce target words; however, it is reasonable to argue that zero should be treated the same as 26, and thus translated as Z. In the case of *alphametics*, the translation A=1, B=2 etc is rarely used; indeed the whole idea is to find the translation of typically 8,9 or 10 digits (ie numbers less than 10) into letters. An example (due to Hunter H Faires) is: find the largest nest such that TEN + EGGS + FIT + SNUG + IN = NEST.

Arithmetic Manipulation

In fact, the numbers may be of interest without manipulation; a near miss is ADIPSY (absence of thirst), which translates into 1,4,9,16,x,25--almost a run of successive squares; and FILO which is simply 6,9,12,15.

Another possibility, still treating each letter separately, is to add, subtract, multiply, divide, raise to a power etc, using a constant.

Number and Placement of Letters Involved in Arithmetical Manipulation

The next possibility is to combine numbers in the source series two, three or more at a time. The investigations in the later articles confine themselves to two letters at a time; indeed, more narrowly, to all adjacent pairs only (leaving the investigation of what happens when the nth number is multiplied by the (n+2)nd and then divided by the square of... to others!).

The obvious decision to be made when selecting adjacent pairs is whether to include the pair formed by the last letter and the first letter. If we do not, our new word will be one letter shorter than the source word, and that is the choice for the later articles. We might call this the choice of *physically adjacent pairs*, as opposed to *all adjacent pairs*. The target number series for ACE would be 2-2 and 2-2-4 respectively.

The simplest operation on two numbers is to add them, and in order not to restrict the process too severely, it will be necessary to count around the end of the alphabet back to the beginning (ie add, then subtract 26 if the sum exceeds 26), eg Y+C = 25+3 = 28, treated as 2=B. The first mention I know of this is by Charles W Bostick (February 1977 "Kickshaws" p 47-8). He listed many 5-letter words and three 6-letter ones: AFFINE, ARPENT and SANDED. In *Making the Alphabet Dance* (1996), Ross Eckler adds the 7-letter CANFULS. These have been called can-do words, since CAN translates to 3,1,14. Addition of adjacent pairs gives 4,15, which re-translates to DO.

We can, of course, use all the letters in a word in a single sum. If we choose to add them, we may find an apt number, such as PAIR = 44 (Susan Thorpe in May 1996 "Kickshaws" p 112). A subset of this occurs when the number is the numerical form of the number itself eg the "word" two-hundred-and-fifty-one = 251 (Dmitri Borgmann); these we may call self-descriptive. In attempting to make the number match its English word, people have used translations other than A=1, B=2, ... (eg E=3, F=9, G=6, ... allows for the numbers from 0 to 12 to represent themselves, as described in "The New Merology" on p 14 of the February 1990 Word Ways). In alphametics, the whole word is treated as a single number.

Thus many different games can be played, according to the method of translation, the number and method of selection of letters, the arithmetic performed, and, as discussed below, whether to operate cyclically or not.

Two types of Subtraction

In attempting to subtract, one very soon encounters a problem. Consider VASE = 22,1,19,5. The differences between adjacent pairs calculate as -21,18,-14, the signs meaning that A is to the left of V, S is to the right of A etc. If we simply ignore the minus signs and take the absolute values of the numbers, we get 21,18,14 which gratifyingly retranslate to URN. We might call this an *absolute difference* method. An alternative with more consistency is to count the distance between letters always in the direction towards the end of the alphabet, so that VASE becomes 5,18,12. It is no accident that this is the same result as obtained by adding 26 to any negative numbers. I call this the *cyclic difference* method. As far as I know, this was first mentioned by Eckler in *Making the Alphabet Dance*. By contrast, the absolute difference method is the original one by Borgmann. When using only physically adjacent pairs, the two types of subtraction produce the same result if all letters in the source word are in ascending sequence, the probability of which increases for shorter words.

Multiple Words

The ideas above can also be applied to multiple words, whether treated as a phrase (Susan Thorpe's FOURTEEN DOZEN = 168, or Borgmann's ONE FOUR SIX = 146), or separately, as in alphametics.

Chains and Trees

Any of the types of translation mentioned may be repeated using the target word as a source, to produce a new target. This in turn may be repeated, to form a *chain*, as in the difference chain BIRDS-GINO-BEA-CD-A, when we may say the chain has a *chain length* of five.

Arranged vertically, with the A at the bottom, we may form a *tree*; for example ACE sits below (is the difference word for) FEHM, NORM and TURM. In this case we may say that ACE has a *fan-out* of three. Arranged in this way, the chain length becomes the *depth of the tree*. (It matters not which way up the tree is drawn; actually trees are most often drawn as root systems, with the single item at the top, but difference trees in mathematics are generally drawn with the single item at the bottom.)