TOP-DOWN SEEMS BEST IN FULL SEARCHES

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In "Designing a List for Word Squares" (August 1996), I showed that a word list intended for finding squares is improved by balancing the beginning and ending letter-frequencies of words. In "Bottom-Up Finds Some Squares Faster" (November 1996) I showed that under certain conditions the old formists' bottom-up strategy is superior to a top-down one. In this article, the concluding one of the series, I examine the use of top and bottom external starter lists for exhaustive search. "External" means that the starter lists do not use any of the computer's valuable 640k lower memory. A starter list is the same as the "gut" list previously introduced, but the words are arranged in a different order.

This study focuses on 7x7 word squares, which provide more extensive statistics to work with. To start, I made a straight-down search of 21,730 OSPD seven-letter words. 7,140 squares were found, indicating a support of 6,117 (the number of words needed to have an expectation of one square resulting). This is 1.55 times as large as Long's theoretical support value of 3,944. Similarly, Richard Sabey in "Which Way to the Square?" found two squares in 26,444 eight-letter OSPD words for a support of 24,200, which is 1.54 times as large as Long's theoretical support value of 15,678. (A support of 22,200 based on Sabey's search of his full Web 2 list is only a little better.) These support values, 50 per cent higher than Long's theoretical one, are consistent with what Long himself found for 4x4 and 5x5 squares.

My working lists for the tests described below consisted of 9,658 words (4/9 of the OSPD). This list size was chosen so that enough squares would be found to be statistically meaningful. By the usual scaling formula $(9,658/21,730)^7 \times 7,140$, 24 squares were expected. 19 were found in one list, 34 in another.

The following table describes the first working list. To read it, 5.3 per cent of the words begin with A, 2.3 per cent end with A, 6.9 per cent begin with B, etc. A top list and a bottom list were developed from this table.

A 5.3, 2.3	H 3.8, 1.5	O 2.7, 0.8	U 2.3, 0.2
B 6.9, 0.1	I 2.2, 0.5	P 7.5, 0.6	V 1.7, 0.0
C 8.9, 1.8	J 1.3, 0.0	Q 0.5, 0.0	W 3.0, 0.3
D 5.5, 11.3	K 1.3, 1.0	R 5.9, 9.5	X 0.1, 0.4
E 3.6, 10.4	L 3.5, 3.0	S 11.5, 32.3	Y 0.4, 6.0
F 4.5, 0.3	M 5.3, 1.3	T 5.9, 5.3	Z 0.5, 0.0
G 4.1, 7.0	N 1.8, 4.1		

By culling all but one of a group of words that are identical except for the final letter, the bottom list for the first test reduced to 9,422 words. Similarly, the top list reduced to 8,581. For the second test, bottom and top list sizes were 8,920 and 8,549, respectively.

Test results are summarized in the two tables below (one on the following page). The data given in the third column is for a straight-down search. It is hard to say what is significant. Bottom-up search found a few squares earlier than top-down. The latter method using the external list found a few squares earlier than the straight-down method. Differences are not large. For search to exhaustion, top-down (or even straight-down) is faster than bottom-up (as we have found all along).

The difference between the two lists (and the number of squares found) is not quite random. My selection process took four words from each series of nine. The first test used the first, third, fifth and seventh; the second used the first, second, sixth and seventh.

top word	bottom word	minutes to find t-d	b-u	str
assists	shaslik	0.2	11.6	2.1
bailers	snarers(2)	15.8	9.2	2.6
costard	dresser	5.0	0.0	7.0
crossed	demesne	1.9	7.5	7.4
dabster	retries	3.5	19.0	7.6
departs	secedes	4.5	-----	8.3
departs	seceded	4.5	4.3	8.3
gardant	tressel	19.5	0.6	12.1
hailers	snarers(2)	-----	9.2	13.2
piefort	tressel	25.2	0.6	21.2
postmen	nelsons	14.4	18.6	21.8
quasses	stodged	0.8	9.7	22.4
remudas	stengah	11.1	16.0	23.7
spathed	deserts(2)	16.7	2.2	27.2
splicer	relents	14.6	23.1	27.4
techier	respect	24.7	8.7	28.9
exhaust	search	29.8	41.3	32.6

For the next two tests, 7,243 gut words were used. One used the fourth, eighth and ninth words from each sequence of nine; the other used the second, third and sixth. Culling produced 6,565 and 6,579 word top lists, and 6,888 and 6,906 word bottom lists. The scaling formula predicted three squares; one test actually found three, and the other, two. The previously-noted fact that the better the word, the longer it takes to evaluate it, is true for both top-down and bottom-up search. The slightly shorter time to exhaust a search with top-down compared to straight-down is due to the culling used with the former.

top word	bottom word	minutes to find t-d	minutes to find b-u
accepts	senates	0.6	10.7
baldric	coarsen	12.9	12.4
bestrew	warstle(2)	12.7	22.8
blatant	tressel	22.6	----
blatant	tressed	22.6	0.4
branded	dessert	19.9	0.1
cascara	arrears(2)	1.7	19.6
cassava	assists	2.9	0.5
cheeped	decease	20.2	7.0
deluded	dessert(2)	22.9	0.1
durmast	tressed(2)	9.5	0.1
erecter	resents	11.0	5.8
espials	shinned	2.4	18.4
espials	sninney	2.4	----
forearm	messier	21.6	2.2
hardset	testees	6.7	6.7
hearsed	dessert	7.1	0.0
mamboed	deadens	16.7	16.2
mandola	artless(2)	25.5	9.0
medaled	dessert	20.0	0.1
parades	sheered	5.8	4.2
repaves	selects	11.7	10.7
strafer	retaste	14.4	10.5
sutures	serenes(3)	15.1	4.1
triseme	erected	13.7	16.4
triseme	erecter	13.7	----
viscera	arrears	4.9	19.6
exhaust search		30.0	39.4

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C A S T L E D
A W E S O M E
S E D A R I N
T S A R I N A
L O R I S E S
E M I N E N T
D E M A S T S
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4.7
2.6
2.1

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O S C U L U M
S C A L E N I
C A L C A R S
U L C E R E D
L E A R I E R
U N R E E V E
M I S D R E W
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4.0 ex at 16.3
4.0 ex at 11.5
7.3 ex at 12.3

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G E S T A T E
E T H A N O L
S H O U T E D
T A U R I N E
A N T I W A R
T O E N A I L
E L D E R L Y
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O U R E B I S
U L U L A N T
R U B A S S E
E L A P I N E
B A S I N A L
I N S N A R E
S T E E L E D
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O U T T R O T
U P H E A V E
T H U N D E R
T E N O U R S
R A D U L A E
O V E R A L L
T E R S E L Y
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Search	GESTATE	OUREBIS	OUTTROT
bot-up	7.1	2.9	3.0 ex at 17.2
top-down	2.3	4.9	7.5 ex at 10.4
str-down	4.2	6.7	6.8 ex at 11.2

For the next pair of tests with seven-letter OSPD words, the gut list of 7,262 words was balanced with respect to beginning letters. The ordered bottom list used precisely the same words as the gut list except for culling. Results for bottom-up and straight-down exhaustive searches are given in the table below.

top word	position in list	time found	bottom word	position in list	time found
cabined	765	2.4	dangled	2226	16.0
carcase	813	2.7	erasers	134	3.4
claimed	938	3.1	decease	173	3.9
coremia	1041	3.6	assists	1	0.0
delimed(2)	1219	4.3	decease	173	3.9
hirsled	2642	8.6	dearest	80	2.3
latened	3476	10.9	deserts	47	1.6
ourebis	5021	15.9	steeled	117	3.2
uraemia	6984	22.1	attests	366	6.6
exhaust		23.0			22.6

Bottom-up search is decidedly advantageous here, as we have seen for 8x8 squares. In this case bottom-up search was actually exhausted in less time than straight-down. The main point is that if the bottom-search had been terminated after four minutes, eight of the ten squares would have been found. Ten squares from 7,262 words indicates a support of 5,224--better than with the full OSPD, but still higher than ideal.

To complete the statistical picture, I extracted 10,853 words from my full Web 2-OSPD list of 37,935 words. Letters in this set were balanced with respect to both beginning and ending words. It took just about the full database to get a fairly complete balance in the extracted list. This set yielded 701 squares in a straight-down exhaustive search, indicating a support of 4,256. Straight-up search of the same list took 28 per cent longer.

I am reluctant to draw too many conclusions from the above studies of 7x7 squares. However, I will say that an exhaustive search of a list large enough to insure finding at least one 10x10 square is impossible. No relevant study so far has indicated that fewer than 250,000 words will be needed. In fact, something like one-and-one-half times 250,000 will be needed unless we produce a balanced list. A heuristic approach must be used. The bottom-up method described here is the only logical choice.

Frankly, I am pessimistic about ever finding a 10x10 square by computer. In my opinion, Eric Albert was lucky to find the single 9x9 square that he did--he found one square using 63,000 nine-letter words, almost the same as Long's theoretical support value of 62,300.