TILING BY COMPUTER

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TILING THE DODECAHEDRON

Making the Alphabet Dance describes on p 214 lettering the 20 vertices of a dodecahedron with 20 different letters so that the five letters surrounding each of the 12 faces can be anagrammed into a word. This comes from an article entitled "Platonic Relationships" in the February 1980 Word Ways. Upon seeing the book, two questions occurred to me. Can I solve this problem with a computer? Are there solutions that do not use the unabridged dictionaries? The answers to both turned out to be yes. The computer program was neither simple nor speedy, but it found plenty of solutions in smaller dictionaries.

Figure 1 is a Schlegel diagram of the dodecahedron. Schlegel diagrams are possible for all the platonic solids and are normally used in this kind of discussion. The circles number the nodes. Eleven of the pentagon faces are numbered. The five outside nodes define the 12th face.

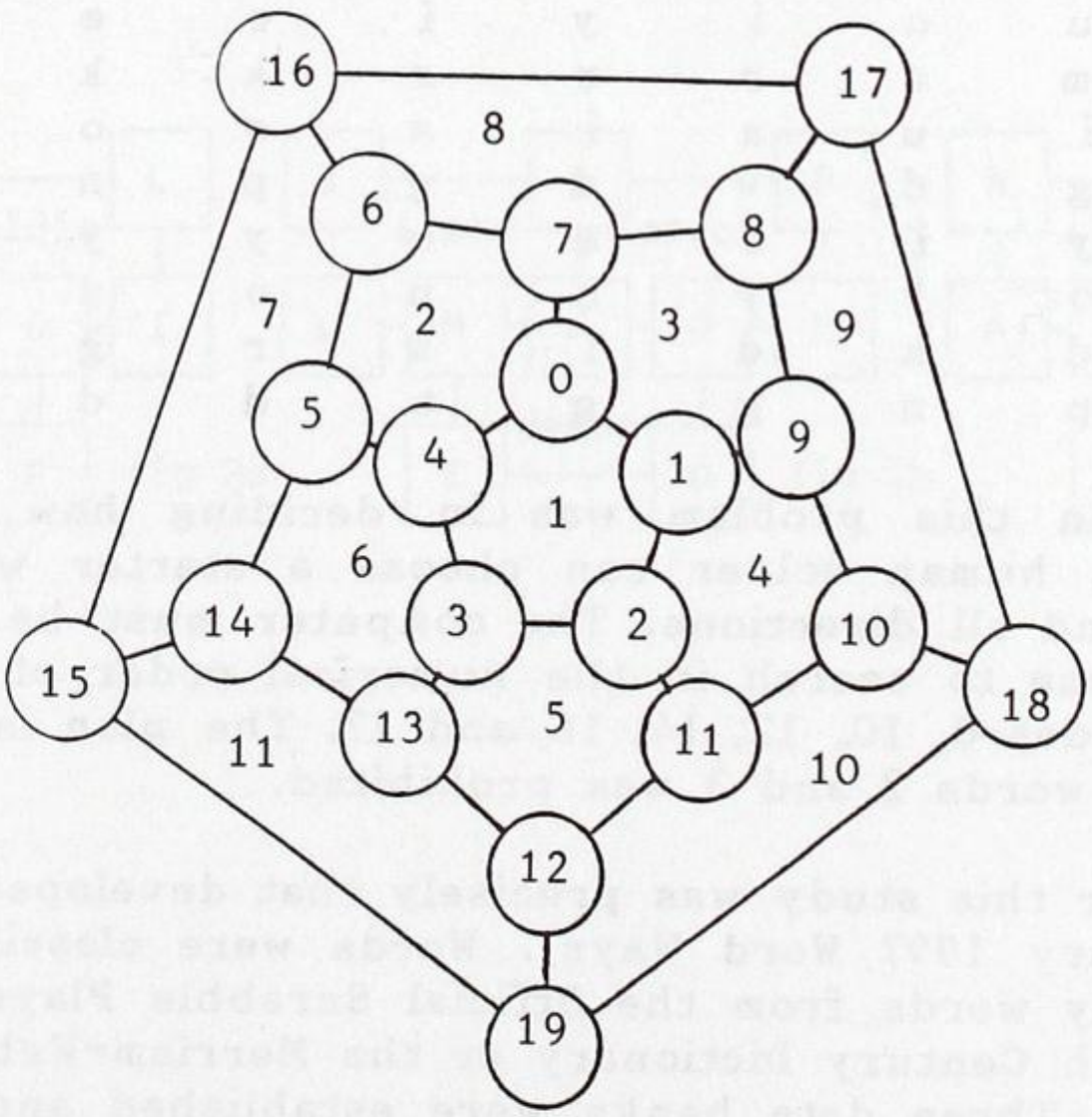

Figure 1

On the next page are nine solutions (in columns). The numbers to the left in the first table correspond to the 12 pentagon faces in Figure 1; the numbers to the left in the second table correspond to the 20 nodes.

1	black	crabs	baths	gambs	botch	botch	becks	bunch	birch
2	jambs	chalk	blank	batch	knobs	knobs	blend	bumfs	blips
3	cramp	thraw	whack	clamp	comps	holms	helms	flung	smith
4	chirk	brown	threw	mewls	mulch	thump	psych	night	thraw
5	whelk	numbs	strum	grows	thirl	crypt	prick	ketch	wreck
6	bowls	micks	limbs	brugh	brant	brick	brank	bocks	block
7	judos	kylix	lying	dhuti	knave	inked	gonad	myops	pylon
8	jumpy	ethyl	conky	typic	pesky	ashen	mould	flamy	spumy
9	gripy	towed	cowed	kelpy	dumpy	haulm	mushy	grail	gamut
10	neigh	pound	drupe	woken	guild	gulpy	tipsy	tried	waged
11	owned	mixup	guimp	round	virga	ridgy	tragi	poked	kendo
12	dungy	pyxed	podgy	dinky	gyved	glade	gusto	pardy	dungy

0	a	a	a	a	o	o	e	u	i
1	c	r	h	m	c	h	s	n	h
2	k	b	t	s	h	t	c	h	r
3	l	s	s	g	t	c	k	c	c
4	b	c	b	b	b	b	b	b	b
5	s	k	l	h	n	k	n	s	l
6	j	l	n	t	k	n	d	m	p
7	m	h	k	c	s	s	l	f	s
8	p	t	c	p	p	h	m	l	m
9	r	w	w	l	m	m	h	g	t
10	i	o	e	e	u	u	y	i	a
11	h	n	r	w	l	p	p	t	w
12	e	u	u	o	i	y	i	e	e
13	w	m	m	r	r	r	r	k	k
14	o	i	i	u	a	i	a	o	o
15	d	x	g	d	v	d	g	p	n
16	u	y	y	i	e	e	o	y	y
17	y	e	o	y	y	a	u	a	u
18	g	d	d	k	d	l	s	r	g
19	n	p	p	n	g	g	t	d	d

The difficulty in this problem was in deciding how the computer should proceed. A human solver can choose a starter word and then continue in any and all directions. The computer must be given explicit instructions. I chose to search in the numerical order of Figure 1 and allow vowels in nodes 0, 10, 12, 14, 16 and 17. The plan is symmetric so exchange between words 2 and 3 was prohibited.

The database for this study was precisely that developed for my Jotto study (see February 1997 Word Ways). Words were classified according to quality and only words from the Official Scrabble Players Dictionary, Chambers Twentieth Century Dictionary or the Merriam-Webster 10th Collegiate were used. Three data banks were established and used appropriately: (1) words with only the chosen starter vowel, (2) words with any one vowel except the starter, and (3) words with two vowels excluding the starter. Working lists came to about 2800 words depending on which vowel was the starter.

The program was very slow-running, so I only found about 100 solutions. A few thousand probably exist. With this database, it is not possible to use Y as the starter vowel. In regard to the consonants, the above examples show the use of J, V and X, although they do not occur often. I did not find any words with Z or Q but this should be possible with words from Webster's Second Unabridged.

OTHER TILINGS

Let's now see what can be done with filling smaller designs with selected letter sets. The letters of PALINDROME will do for a starter. Its ten letters allow spelling 110 five-letter words. Figures 2a, 2b and 2c each show the use of ten letters to define three interlocked pentagons.

The 110 words can fill one of these figures in no fewer than 5181 ways. So, to be more selective, let's require that the eight letters surrounding each pair of pentagons spall a word as well. Turning to Darryl Francis's article "Palindrome and Palindromes" in the August 1989 Word Ways, we find several trans-2-deletions of PALINDROME. Six of these are in my database. Using them, there are only ten ways to fill a figure. Several are minor variants of the following three, shown in Figures 2a, 2b and 2c. The eight-letter words used in these examples are POMANDER, MELANOID, IMPLORED and PROLAMIN.

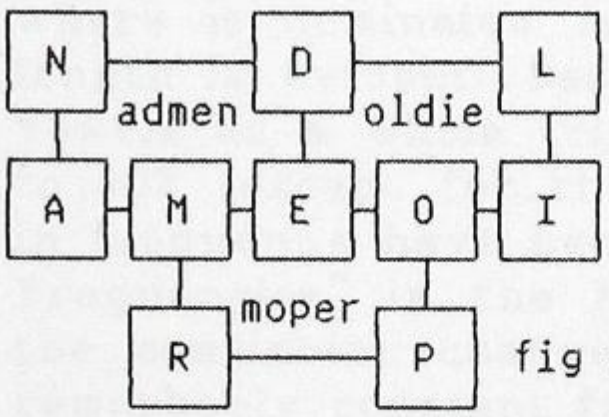

fig 2a

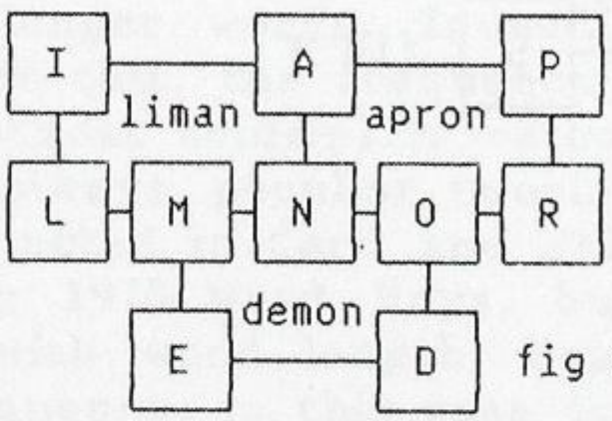

fig 2b

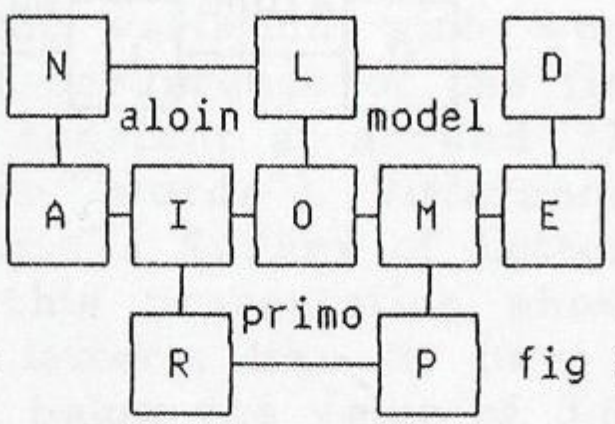

fig 2c

In ''Substitute-Letter Isogram Networks'' in the August 1993 Word Ways, recognize that PALINDROME subtransposes to IMPERSONAL and there are three more words one more step away in the network on the top of p 160. Two words, PORCELAINS and ANGIOSPERM, are in the 10th Collegiate. Figures 3a and 3b illustrate sub-2-transposals. NEUROPLASM is in Chambers. Using it allows the two-step sub-2-transposal chain of Figure 4. Presumably the reader can identify the third word!

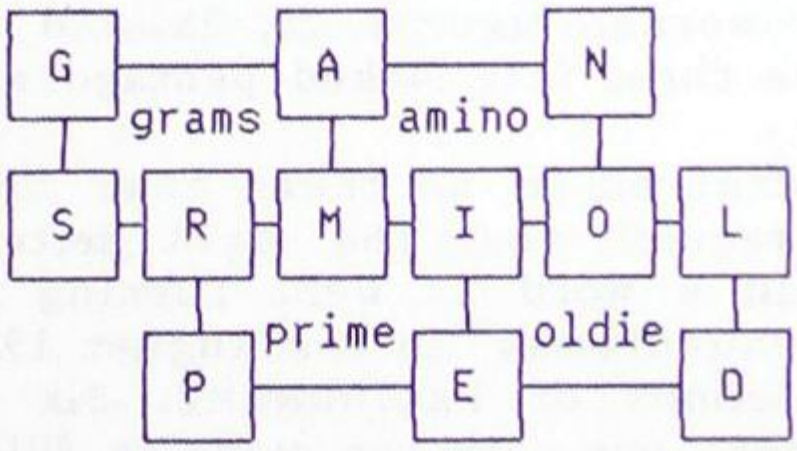

fig 3a

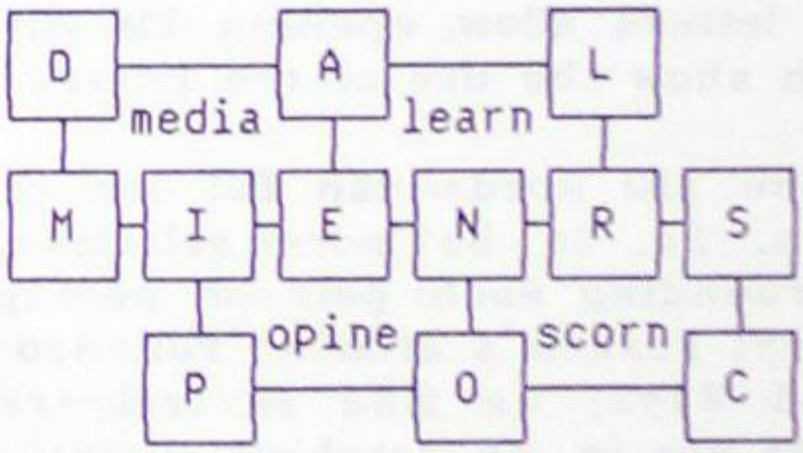

fig 3b

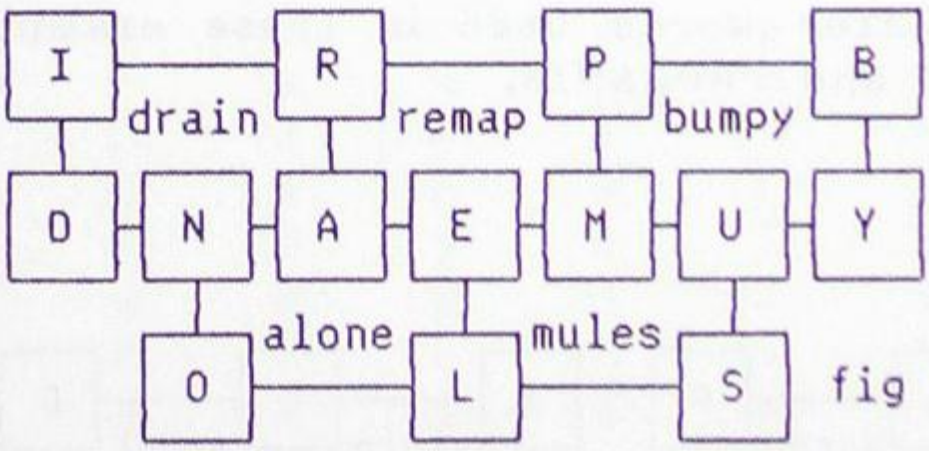

fig 4