F. P. RAMSEY, MEET SCRUBWOMAN EDITH

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F.P. Ramsey (1903-1930) was a philosopher, logician, and mathematician who died young after an ill-fated operation. He was a product of England's Cambridge University when both Bertrand Russell and Ludwig Wittgenstein were there. According to the Cambridge Dictionary of Biography, he was among the first to appreciate and criticize Wittgenstein's Tractatus, and, rejecting the idea of ineffable metaphysical truth in that work, remarked "what we can't say we can't say, and we can't whistle it, either."

Ramsey theorems in combinatoric mathematics are existence proofs that in simplest form are variously called the pigeonhole principle, the Dirichlet drawer principle, the shoe-box argument, or the theorem of the boxes. This theorem states that if you have at least n+1 marbles and place them in n boxes, then at least one box has at least two marbles in it. For example, if there are thirteen people then at least two will have a birthday in the same month.

The late Paul Erdös did much to popularize Ramsey theory with problems similar to the following: What is the smallest number, n, of people for which we are assured that among the n, there are either three persons who are mutual friends or there are three persons who are complete strangers to each other? This minimal number is called the Ramsey number of the problem, and in this case it is known to be six. (See Ronald L. Graham, Bruce L. Rothschild, Joel H. Springer, Ramsey Theory, Wiley 1990.)

Our new two-person word game is ultimately based on Ramsey theory. Take the fifteen letters in SCRUBWOMAN EDITH and place them singly on fifteen small tiles (we call these scrub tiles, but resist the temptation to call the game Scrubble).Prepare also the following list of twenty words for the use of both players:

BAN	MOC	BDS	CWT
CHI	SEA	CUR	DEN
DIM	TAU	HOD	WAR
HUB	MEW	MRS	NTH
RIB	TOE	SOU	WIN

All twenty words are main entries in Webster's Second except for BDS (Bachelor of Dental Surgery) which occurs in the Abbreviation section, and MOC (short for moccasin) which occurs in Webster's Third. Notice

that each letter occurs in exactly four words and any two letters occur together in at most one word.

There are two versions of the game. The first is played by turning the scrub tiles face down and the players will alternately draw a scrub and turn it face up. The first to form a word on the Ramsey list wins. It may not be obvious, but there can be no ties in this game.

The second version is more challenging and starts with all fifteen scrubs face up to be used by both players. They alternately draw a scrub of their choice and, as before, the first to be able to form a Ramsey word wins. It seems to be a new result that this version is a forced win for the first person (even if the first draw by each player is made at random).

Our word game is usually played as an edge-coloring game on the complete hexagon graph shown the next page. For clarity in the diagram, we have not labeled the edges; instead, any two nodes (say TOUCH and BURSA) are edge-joined by a common letter (here U). Two players in the geometric game alternately color an edge red or blue, and the first player to form a triangle of his color wins. In the word game, all possible triangles form one of the twenty words on the Ramsey list. Either version is equivalent to the former Ramsey problem of the three mutual friends or three mutual strangers. The fact that the Ramsey number is six means that there can be no ties in these games.

For the word game where players may choose their scrubs, we must, to win as first player, be able to force the second player to waste moves as we prepare a double threat. This requires us to be able to locate the letters that "miss" his choice of letter in word formation. Every letter of SCRUBWOMAN EDITH does not occur in one of the twenty words with exactly six other letters. For example, the letter H does not occur with any of the letters RSWAME in the word list. Equivalently, H misses the words SEA, MEW, WAR and MRS in the list. Our Ramsey word list is not written down at random, but is designed to locate the four words missed by each letter. The words in that list are paired with words that they miss. The pairs occur in the first and second columns as well as the third and fourth columns. For instance, CHI is paired with SEA which means that every letter in one of these two words misses every letter of the other--H misses, for example, each letter in SEA. To find all the letters that H misses, locate aoll four appearances of H (CHI, HUB, HOD, NTH) and note the four words SEA, MEW, WAR and MRS paired with them.

As an example of the use of this information, suppose the first player chooses R, and the second H. As first player, we must find a letter that forces the second player to waste his next move. For instance, if we choose W (threatening to complete WAR in one more move), the second must play A to block and this does not help him. We can now play, say, C and have a double threat (CWT or CUR).

A
NEWT

TOUCH

WC
RIM

H
BIND

BURSA

DOMES