DOUBLY TRUE ALPHAMETICS IN BASE 27

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In "Base 27: the Key to a New Gematria" in the May 1993 Word Ways, Lee Sallows offered "immortal fame" for finding doubly true equations (those valid in both base 10 and base 27 notation). I do not know where he obtained authority to grant such fame, but I accept his challenge.

The reason Sallows did not find any doubly true equations is probably because a brute-force computer search is impractical; wints corresponding to even modest number-names in base 27 are likely to be very large, in the millions or greater. (Recall that a wint is defined as a base 10 number which corresponds to a word in base 27 notation.) To keep wint sizes down, I introduce paired wints, such as SEVEN - THREE = -587565 or FIFTY - FORTY = -126846. In particular, I let the computer search for values of n1, n2, ... n50 satisfying the following equations:

Base 27 n1(11318) + n2(15216) + n4(129618) + n5(125258) +
  n6(14118) + n9(282506) + n10(14729) + n7(-587565) + n50(-126846) = 0
Base 10 n1(1) + n2(2) + n4(4) + n5(5) + n6(6) + n9(9) + n10(10) +
  n7(4) + n50(10) = sum

where, in base 27, ONE = 11318, TWO = 15216, FOUR = 129618, and so on.

The computer searched for those n1, n2, ... n50 which satisfy the first equation, letting the base 10 sum of the second come out as is. Table 1 lists coefficients for a few of the solutions, found in about an hour's run. The coefficients for equation 7 create the only doubly true alphametic (base 10 sum of zero), but one can generate additional doubly true alphametics by adding or subtracting the others, as shown in Table 2.

Note that none of the equations in Table 2 have zeros for both n7 and n50. This tells us that doubly true relationships using only the three-letter and four-letter number names are probably impossible to find. Using unpaired five-letter number names forces us to deal with horrendously-large wints, increasing the search time.

Other paired wints could have been used: FIFTY - EIGHT = 531041, FORTY - EIGHT = 657887, SIXTY - SEVEN = 80606 or TWENTY - TWELVE = 1424. Using wint pairs with a common coefficient and then rearranging the final equation can disguise the method and give the impression that the computer worked with giant numbers.

table 1

n1	n2	n4	n5	n6	n9	n10	n7	n50	sum	eqn
-5	-4	+1	-2	+1	0	-2	0	-2	-53	01
-3	+4	+5	-4	-5	+3	-2	+2	-2	-30	02
-2	-5	-2	0	-1	+3	-1	+1	-1	-15	03
-6	+3	+2	-2	+3	-1	0	0	-2	-13	04
-1	-5	0	+1	+2	+1	-1	+1	-2	-11	05
-1	+3	-5	+4	-1	0	0	0	-1	-11	06
-3	-4	0	-1	-6	+2	+5	+1	-2	0	07 *
-5	0	-2	+6	-4	-2	+4	0	-1	5	08
-2	-4	+3	+4	+5	-3	-2	0	0	5	09
-4	+5	-6	+4	0	+2	+1	+1	-2	14	10
-4	-3	+3	+2	-3	-2	+4	0	0	16	11
-3	-3	+5	+3	0	-4	+4	0	-1	20	12
-3	-6	+4	-3	+4	+3	0	+2	-2	25	13
-3	+2	-1	0	+1	+2	+1	+1	-1	25	14
-6	-2	0	+3	+3	+1	-1	+1	0	26	15
-5	-2	+2	+4	+6	-1	-1	+1	-1	30	16
-5	-3	-4	-6	+4	+4	+4	0	-1	33	17
-6	-1	+4	+3	+1	-2	+5	+1	-2	45	18

table 2

n1	n2	n4	n5	n6	n9	n10	n7	n50	sum	eqns
-3	+4	-5	+2	-9	+1	+6	0	-1	0	8 -9
0	-8	+5	-3	+3	+1	-1	+1	-1	0	13-14
-8	+2	+7	0	+1	+2	-3	+3	-3	0	2+16
0	-8	+5	-3	+3	+1	-1	+1	-1	0	5 6
-11	+6	+2	+2	-8	+3	+3	+3	-4	0	2+10+11
-11	-2	+7	-1	-5	+4	+2	+4	-5	0	2 +3+18
-13	-10	+2	-5	+5	0	+6	0	-4	0	1+12+17
-18	+4	+4	-1	+9	-1	-1	+1	-4	0	4 +4+15