WORD LADDER SQUARES AND CUBES

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In its traditional form, the word ladder is a one-dimensional structure; even word ladder networks with many branchings proceed in a basically linear fashion. But one can conceive of word ladders which extend solidly in two and three dimensions, in the manner of the familiar word square and word cube. This article will suggest some definitions and rules of construction which might reasonably apply to such arrays, and offer some illustrative examples.

DEFINITIONS

If the two end words of a word ladder have no letters in common at the same positions, and if the ladder contains only the minimum number of steps needed to completely transform words of that length, the ladder is known as an ideal word ladder (example TIME-TIRE-WIRE-WARE-WARP). Ideal word ladders are the basis for the classification scheme which follows.

If in a square array of words ("square" in the sense that its rows and its columns have the same number of words) every row and every column is a word ladder, the array is a word ladder square (WLS). The three-dimensional counterpart of the WLS is the word ladder cube (WLC). Other two- and three-dimensional word ladder shapes may exist, of course, but these will not be considered here. Also excluded from consideration will be squares or cubes in which word ladders may be deemed to be anything other than single rows or columns. The suggested minimum "regulation" size for a WLS is a square of side four, and for a WLC it is a cube of edge four. Further definitions:

Simple WLS or WLC: one in which there are no ideal word ladders

Ideal WLS: one in which at least one, but not all, of the word ladders is an ideal word ladder

Perfect WLS: one in which every word ladder is an ideal word ladder

Ideal WLC: one in which at least one, but not all, of the constituent WLSs is ideal

Perfect WLC: one in which at least one, but not all, of the constituent WLSs is perfect

Sublime WLC: one in which all of the constituent WLSs are perfect

In addition, it is important to distinguish between what might be called proper and improper word ladder arrays. The latter are essentially trivial arrays which are easily produced by the systematic repetition of segments of word ladders; as such, they cannot be considered logologically challenging, although they do possess a number of points of interest. The rule which distinguishes proper from improper arrays will be described in the next section.

Heterogeneous word ladder arrays are those in which all of the words are different. Proper WLSs and WLCs may be either heterogeneous or nonheterogeneous.

Lastly, a standard word ladder array is one which contains no ladders in which the same word appears more than once, and a nonstandard array is one which contains at least one such ladder. Except where otherwise noted, the word ladder arrays discussed in this article are all standard arrays.

PROPER WORD LADDER SQUARES AND CUBES

The Two-Word Sequence Rule A given word may appear more than once in both proper and improper WLSs, but in a proper WLS the same two-word sequence (not combination) may not appear more than once in the rows and once in the columns. For example, in a proper WLS the sequence TALL-TALE, if it appears in a row, may not appear in any other row, but it may appear in one column; the sequence TALE-TALL, however, being considered a different sequence, may appear once in the rows and once in the columns of the same square. In a proper WLC, this rule applies to the individual constituent WLSs but not to the WLC as a whole. The reason for choosing this particular rule (which I call the "two-word sequence rule") to distinguish between proper and improper WLSs and WLCs will be discussed in the section on improper word ladder squares and cubes.

Three-Letter Words Ideal and perfect WLSs of three-letter words are easily found. Of the four which follow, the first is ideal and the rest are perfect:

CAT COT COG DOG	FAN PAN PEN PET	RAT BAT BOT BOG	RIP REP REX VEX
CAD COD COT DOT	FIN PIN PUN PUT	RET BET BIT BIG	RID RED RET VET
COD mod mot ROT	FIG PIG PUG PUS	RED BED BID BIN	ROD RAD RAT VAT
GOD ROD ROT RAT	DIG BIG BUG BUS	WED FED FID FIN	SOD SAD SAT CAT

In these and the other WLSs and WLCs in this article, words which are a part of one or more ideal word ladders are shown in upper case letters, and words which are not a part of an ideal word ladder are shown in lower case letters.

Ideal and perfect proper WLCs for three-letter words are not difficult to construct, although care must be taken that all of the 12 WLSs in the WLC conform to the two-word sequence rule. Only a third of these WLSs need be shown to define the cube, however; for example, the following four WLSs, if placed atop one another in order from left to right, define an ideal proper WLC:

MAN MAD GAD GOD	MAT mar mad mod	BAT bar bad mad	fat far fad dad
CAN CAD SAD SOD	CAT car cad cod	TAT tar tad cad	gat gar gad lad
CAR cab cad COD	CAP can cab COB	TAP tan tab CAB	gap gan gab LAB
CUR car cap cop	CUP cap cat cot	TUP tap tat cat	TAP tan tab cab

It is probably impossible to form a sublime proper WLS of three-letter words (or of words of any other length). Although the next WLC is shown in all upper case letters, it is not a sublime WLC, as only the four of its 12 constituent WLSs that are displayed are perfect; it is thus merely a perfect proper WLC.

FAN PAN PIN PIT	GAN BAN BIN BIT	CAN WAN WIN WIT	BAN TAN TIN TIT
FIN PIN PUN PUT	GIN BIN BUN BUT	GAN BAN BIN BIT	RAN FAN FIN FIT
FIG PIG PUG PUN	GIG BIG BUG BUN	GAG BAG BIG BIN	RAG FAG FIG FIN
BIG RIG RUG RUN	WIG DIG DUG DUN	GIG BIG BUG BUN	RIG FIG FUG FUN

Four-Letter Words Simple proper WLSs of three- or four-letter words are generally very easy to create, but may be less so if the object is to incorporate interesting or related words at the corner (and perhaps also the center) positions. An example of such a corner- and center-oriented simple WLS is shown below; the two squares in the next row are perfect proper WLSs.

love	hove	have	hake	hate
lave	have	hive	hike	hake
wave	lave	live	like	lake
gave	cave	lave	lake	wake
give	gave	rave	rake	take

WORD	LORD	LORE	LONE	LANE
CORD	BORD	BORE	BONE	BANE
CARD	BARD	BARE	BANE	BINE
CARE	BARE	BARD	BAND	BIND
CANE	BANE	BAND	BARD	BIRD

BANK	BARK	BARD	BORD	CORD
BASK	BALK	BALD	BOLD	COLD
BASE	BALE	BALL	BOLL	COLL
BISE	BILE	BILL	BELL	CELL
WISE	WILE	WILL	WELL	TELL

Ideal and perfect proper WLCs of four-letter words are somewhat tricky to devise, mainly due to the need to satisfy the two-word sequence rule in the 15 constituent WLSs of such cubes. At the top of the next page are the exploded views of two proper WLCs of four-letter words: the one in the left-hand column is ideal, and the one in the right-hand column is perfect.

RIDE BIDE BODE BONE BOND
RODE BODE BADE BANE BAND
MODE CODE CADE CANE bane
MOLE COLE CALE CARE bare
MOLL COLL CALL CARL barl

RILE BILE BOLE BORE BORD
ROLE BOLE BALE BARE BARD
mole cole cale care bare
pole mole male mare care
poll moll mall marl carl

sile mile mole more bore
sole mole male mare bare
tole pole pale pare mare
role bole bale bare hare
roll boll ball barl harl

tile vile vole mole bole
tole vole vale male bale
hole sole sale pale male
dole tole tale bale hale
role bole bale ball hall

tole vole vile mile bile
tale vale vole mole bole
hale sale sole pole mole
dale tale sale pale male
rale bale pale pall mall

FURS CURS CARS CART CANT
FURL CURL CARL CARE CANE
FULL CULL CALL CALE CADE
FELL CELL COLL COLE CODE
BELL MELL MOLL MOLE MODE

FURL CURL CARL CARE CANE
curl burl barl bare care
cull bull ball bale cale
cell bell boll bole cole
yell dell doll dole mole

FULL CULL CALL CALE CAPE
cull bull ball bale cale
hull mull mall male pale
hell mell moll mole pole
well tell toll tole sole

BULL MULL MALL MALE cale
dull cull call cale rale
mull hull hall hale sale
mell hell holl hole sole
sill bill ball bale dale

BELL MELL MOLL MOLE cole
dell cell coll cole role
mell hell holl hole sole
mill hill hall hale sale
sill bill ball bale dale

Five- and Six-Letter Words Here are two ideal proper WLSs of five-letter words and one ideal proper WLS of six-letter words:

HONOR HONER LONER LINER LIVER LIVED
HONER hoser loser loner lover loved
HOVER hoper loper lover lower lowed
DOVER doper coper cover cower cowed
DIVER dover cover corer coder coded
DIVES doves coves cores codes codex

NAVAL NAVEL RAVEL RATEL RATES RITES
NAVEL naves rates rates bates bites
RAVEL raves paves pates mates mites
RAVED raver paver pater mater miter
ROVED rover raver rater cater mater
ROBED roved raved rated rater water

VENDOR VENDER VENTER VESTER VESTED TESTED TASTED
vender render renter rester rested bested basted
mender tender tenter tester tested jested bested
mended tended tented tested tester jester bester
tended rended rented rested rester vester fester
tented rented vented vested vester wester pester
tested rested vested nested nester fester jester

As may be seen from these examples, the simplest (and perhaps only) way to make ideal proper WLSs of five- and six-letter words is to use mostly four-letter words with one- and two-letter suffixes added. It may

not be possible to devise a perfect proper WLS for five-letter words, and it is very probably impossible to devise one for six-letter or longer words. Likewise, an ideal or perfect proper WLC of five-letter words may be impossible to construct, and one of six-letter or longer words is very likely so.

Patterned Simple Proper Word Ladder Squares and Cubes In contrast, simple proper WLSs and WLCs of words seven or more letters long can readily be made. An easy way to do this is to take groups of words which differ from each other at only one or two letter positions and arrange them in accordance with certain general matrix patterns which comply with the two-word sequence rule. The pattern at top left below, for example, generates a simple proper WLC from any four words of the same length which differ from each other at only one letter position; the pattern at top right does the same from any six words of this kind. Any one of the individual matrices in these WLC patterns may also be a pattern for a simple proper WLS, as exemplified by the 36-word WLS at bottom.

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1234 2413 3142 4321
2413 4321 1234 3142
3142 1234 4321 2413
4321 3142 2413 1234
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123456 246135 362514 415263 531642 654321
246135 415263 654321 123456 362514 531642
362514 654321 246135 531642 123456 415263
415263 123456 531642 246135 654321 362514
531642 362514 123456 654321 415263 246135
654321 531642 415263 362514 246135 123456
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dustinesses fustinesses gustinesses lustinesses mustinesses rustinesses
fustinesses lustinesses rustinesses dustinesses gustinesses mustinesses
gustinesses rustinesses fustinesses mustinesses dustinesses lustinesses
lustinesses dustinesses mustinesses fustinesses rustinesses gustinesses
mustinesses gustinesses dustinesses rustinesses lustinesses fustinesses
rustinesses mustinesses lustinesses gustinesses fustinesses dustinesses
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Words of an even wider wingspan (all from Webster's Third) flock together in this patterned simple proper WLS of sixteen 15-letter words:

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lightlessnesses nightlessnesses rightlessnesses sightlessnesses
nightlessnesses sightlessnesses lightlessnesses rightlessnesses
rightlessnesses lightlessnesses sightlessnesses nightlessnesses
sightlessnesses rightlessnesses nightlessnesses lightlessnesses
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Matrix patterns for generating simple proper WLSs and WLCs of other sizes exist, as do similar patterns for groups of words which differ at two letter positions. These rather monotonous arrays are of interest chiefly because they permit relatively long words to be used in proper WLSs and WLCs.

Heterogeneous Word Ladder Squares and Cubes Heterogeneous word ladder arrays are a variety of proper word ladder array in which no two words may be the same. Owing to their nonredundant character they are perhaps the most esthetically pleasing of the word ladder arrays,

but for the same reason they are also relatively difficult to devise and somewhat restricted in scope. A number of the ideal and perfect WLSs of three- and four-letter words shown previously have been heterogeneous, but it seems doubtful that any ideal or perfect heterogeneous WLS of five-letter or longer words could be formed. Entirely unexplored is the question of how large the largest possible simple heterogeneous WLSs of various word lengths can be. Larger simple heterogeneous WLSs of four-letter words than this 100-word specimen, for instance, can doubtless be assembled--but just how much larger?

wire hire dire dice pice pile wile wine sine site
ware hare dare dace pace pale wale wane sane sate
fare pare mare mace race rale gale gane dane date
fane pane mane mate rate rave gave game dame dale
vane hane bane bate late lave cave came tame tale
vale hale bale bade lade lane cane cake take tare
vole hole bole bode lode lone cone coke toke tore
tole mole role rode node none pone poke moke more
tile mile rile ride nide nine pine pike mike mire
vile sile bile bide tide tine bine bike like lire

An eleventh column, cite-cate-fate-fale-cale-care-core-sore-sire-fire, could be added to this square on the right, but then it would no longer be conceptually square, having become a rectangle in plan as well as appearance.

Heterogeneous WLCs are feasible, but are considerably more difficult to devise than nonheterogeneous WLCs. Only at the cost of including a disagreeably-large number of exotic words, for example, was I able to complete the following simple heterogeneous WLCs of three-letter and four-letter words:

cat nat rat wat	bat fat gat tat	bar far gar tar	car mar par war
cag nag rag wag	bag fag gag tag	ban fan gan tan	can man pan wan
cay nay ray way	bay fay gay tay	bad fad gad tad	cad mad pad wad
jay kay may pay	day hay lay say	dad had lad sad	rad vad yad zad

bade cade fade made
bale cale fale male
hale rale sale wale
hake rake sake wake

bake cake fake make
bane cane fane mane
hane rane sane wane
have rave save wave

bare care fare mare
bate cate fate mate
hate rate sate wate
hage rage sage wage

bars cars fars mars
bats cats fats mats
hats rats sats wats
hags rags sags wags

It is probably possible to form simple heterogeneous WLCs of four-letter words in which all of the words can accept identical one- to four-letter suffixes; if it is, simple heterogeneous WLCs of words up to eight letters in length can obviously then be made from it. It is also possible

(barely) that ideal or perfect heterogeneous WLCs of three- or four-letter words can be fashioned.

IMPROPER WORD LADDER SQUARES AND CUBES

Improper word ladder arrays contain repeated segments of the same word ladder in their rows and/or columns, and may in fact be entirely composed of such repetitive segments. Given the word ladder PASS-PANS-PANT-PINT-DINT-DIRT-DIRK-DARK-HARK-HANK-HAND-HIND-FIND one can simply write down progressively offset segments of the ladder to obtain the perfect WLS at left below. A similar procedure generates from this WLS the WLC shown at right below. To save space, the cube is depicted symbolically, with the letters A through M representing, in order, the words in the PASS/FIND ladder.

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PASS PANS PANT PINT DINT        ABCDE BCDEF CDEFG DEFGH EFGHI
PANS PANT PINT DINT DIRT        BCDEF CDEFG DEFGH EFGHI FGHIJ
PANT PINT DINT DIRT DIRK        CDEFG DEFGH EFGHI FGHIJ GHIJK
PINT DINT DIRT DIRK DARK        DEFGH EFGHI FGHIJ GHIJK HIJKL
DINT DIRT DIRK DARK HARK        EFGHI FGHIJ GHIJK HIJKL IJKLM
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It may be worth noting that the cube thus generated is an example of a sublime WLC, in which all 15 WLSs are perfect and all 75 word ladders are ideal. Not every word ladder will generate a perfect WLS and a sublime WLC in this manner; only those which incorporate cycles in which each letter position changes once per cycle, and the changes are in the same order in each cycle, will do so. In the PASS/FIND ladder, for instance, the sequence of letter position changes is 3421, 3421, 3421. It might also be mentioned that it is possible to generate improper WLSs and WLCs with fewer different words by utilizing circular word ladders. As an example, the circular ladder BANE-BONE-CONE-CORE-CORD-CARD-BARD-BAND-BANE will generate a perfect WLS and a sublime WLC with only eight different words, as compared to the PASS/FIND ladder's total of thirteen.

Logologically, there is an evident need to distinguish such automatic constructions as these from their non-trivial counterparts, the proper word ladder squares and cubes; the question is what exactly that distinction should be. An esthentically attractive alternative to the two-word sequence rule might be to require that proper word ladder arrays be heterogeneous. But such a constraint would, I believe, be severely and unnecessarily limiting. On the other hand, the construction of proper WLSs and WLCs could be made much easier if some more lenient rule which, say, prohibited only three-word sequences from appearing more than once in the rows or columns of a proper WLS, or one which allowed a two-word sequence to appear twice but not three times in the rows or columns of a proper square, were to be adopted. Alas, such arbitrary rules as these have a decidedly unesthetic feeling to them. To me, the rule which permits any two-word sequence to occur only once in the rows and only once in the columns of a proper WLS combines a reasonable esthetic appeal with an acceptable degree of difficulty. But it

is not the only rule available, nor is it necessarily the one best suited to every taste.

NONSTANDARD WORD LADDER SQUARES AND CUBES

As previously noted, nonstandard word ladder arrays are those in which a word may appear more than once in the same ladder. The greater latitude afforded by such arrays may serve a variety of creative purposes, such as facilitating the arrangement of key words in a WLS (first example below), enabling the construction of WLSs in which all of the ladders are circular (second example below), and permitting the fabrication of WLSs in which all of the ladders are palindromic:

checker chicker chucker clucker plucker
chicker whicker chicker chucker clucker
chocker chicker chicken chicker clicker
chucker chocker chicker clicker clocker
shucker shocker chocker clocker crocker

corner conner canner conner corner
conner canner canter canner conner
canner canter center canter canner
conner canner canter canner conner
corner conner canner conner corner

keel keep peep peek leek
keep neep peep peen peek
peep peep peep peep peep
peek peen peep neep keep
leek peek peep keep keel

leper lever rever revel repel
lever rever refer rever revel
rever refer reder refer rever
revel rever refer rever lever
repel revel rever lever leper

Nonstandard WLCs having the same properties may also be devised. An example of a fully palindromic WLC, in which all of the 75 ladders are palindromic, is displayed below. Since the cube is fully palindromic, it is adequately defined by showing just the first three squares in a stack; the fourth and fifth squares in the stack are obtained by simply reversing the individual words in the second and first squares, respectively.

got not tot ton tog gut nut tut tun tug tut tut tot tut tut
pot tot tut tot top put tut tat tut tup tut tat tit tat tut
tot tut tat tut tot tut tat tot tat tut tat tit tut tit tat
top tot tut tot pot tup tut tat tut put tut tat tit tat tut
tog ton tot not got tug tun tut nut gut tut tut tot tut tut

PARAMETERS UNKNOWN

As broad as this overview of word ladder squares and cubes has been, it is still far from being a complete survey of the entire field of word ladder arrays. Omitted from the discussion have been such topics of potential logological interest as arrays which are not square or

cubical in shape, arrays which are not orthogonal in structure, word ladder hypercubes, arrays in which word ladders are not confined to a single row or column, and arrays in which the letters in the words have been assigned numerical values.

With respect to the topic with which this article has been primarily concerned, the standard proper word ladder squares and cubes which are governed by the two-word sequence rule, there will no doubt always be a number of more or less open questions relating to maximal dimensions. At the moment, these include the following:

1. Can a regulation-sized simple proper WLS of words longer than 15 letters be formed?
2. Can an ideal proper WLS of words longer than six letters be formed?
3. Can a perfect proper WLS of words longer than four letters be formed?
4. What are the largest simple proper WLSs of different word lengths which can be formed?
5. Can a regulation-sized simple proper WLC of words longer than 15 letters be formed?
6. Can an ideal proper WLC of words longer than four letters be formed?
7. Can a perfect proper WLC of words longer than four letters be formed?
8. Can a sublime proper WLC of any word length be formed?
9. What are the largest simple heterogeneous WLSs of different word lengths which can be formed?
10. Can any ideal or perfect heterogeneous WLSs of words of five or more letters be formed?
11. What are the largest simple heterogeneous WLCs of different word lengths which can be formed?
12. Can any ideal, perfect or sublime heterogeneous WLC be formed?

Clearly, much of this terrain remains terra incognita.

REFERENCES

All of the words used in the WLSs and WLCs in this article are dictionary-listed, and may be found in one or more of the following: Webster's Third New International Dictionary, the Oxford English Dictionary, Webster's New International Dictionary Second Edition ('chocker'), and the New Dictionary of American Slang ('hoser').