STUFFING THE ISOGRID

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With the gunpowdery blacksmith at our side, we have hunted the biggest bisograms. In our next adventure, they will be stuffed and put on display. The "Nots and Crosses" contest in the July 1991 issue of Games Magazine put forth the following challenge: place a different word in each cell of a 3-by-3 tic-tac-toe grid ("noughts and crosses" for Anglophiles) so that no row, column, or corner-to-corner diagonal repeats a letter, with the object being to maximize the total number of letters in the grid. Word acceptability is governed by standard Games Magazine contest rules, allowing solidly-written, uncapitalized boldfaced entries in the main section of Webster's Third (W3), and clearly-implied inflected forms. If no row, column, or diagonal repeats a letter, it is clear that no individual cell can repeat a letter; thus each word in the grid must be an isogram (more precisely a "solo isogram"). Our grid will be termed an *isogrid*, and we seek to stuff it with as many letters as possible. To whet the reader's appetite, here is the sample grid given by Games

shmaltz	upwind	very	1 2 3
wider	fog	humpbacks	4 5 6
puck	blathers	jinx	7 8 9

with cell numbering at the right. The problem will be analyzed with the goal of proving that the maximum score achieved in the Games contest is in fact the best possible, given certain assumptions about known words. This will determine a set of constraints which would need to be met for a better score to be found. Those who wish to try the challenge themselves may want to stop reading here and break out pencil and paper!

THE BASICS

We start by identifying some basic terminology and obvious facts about the isogrid. As shown above, cells are numbered 1 to 9 from top left to bottom right. Cells which share a letter are said to "crash". Cells which are in the same row, column, or diagonal are said to "see each other". Cells 2,4,6,8 are edge cells; 2 and 8 form one edge pair, and 4 and 6 the other. The remaining cells--1,3,5,7,9--form the checkerboard, in which each cell sees all the others. Each edge pair is a bisogram--a pair of words which do not repeat a letter. Analogously, the checkerboard constitutes a quintisogram. A set of N isogrammatic words totaling L letters will often be referred to as an (NxL), pronounced "N by L". Two previous Word Ways articles have covered the search for long bisograms and quintisograms, and have assembled a reference list of each to be used as ammunition in this challenge.

Each letter can be used a maximum of three times in the isogrid. If a letter is used in the center, it cannot be used anywhere else, since all cells in the grid see the center. Since each individual letter is acceptable as a word under the rules of the contest, it is very possible we will want only one letter there. If a letter is used in a corner cell, it can also be used in each of the two non-adjacent edge cells. For example, a letter could be used in cells 1,6 and 8.

We can break down the task of "stuffing the isogrid" to the subtasks of finding bisograms to form the edge pairs which can interlock, in a sense, with the quintisogram that forms the checkerboard, while maximizing the total lengths of each. An isogrid can be categorized by the lengths of its two edge pairs (longest first), and its checkerboard. For example, the sample grid above would be a (14,14,23).

STARTING FROM THE TOP

A natural starting point is to determine the maximum theoretical score, assuming any set of letters could constitute a word. We might imagine a theoretical grid which would score (26,26,26) = 78. However, at least one letter must appear in the center cell, and thus could not appear in either of the edge pairs, making the theoretical best grid (25,25,26) = 76.

We can construct such a grid in the following way. In order to fill each of the cells in the checkerboard without crashing, we can place the letters ABC...V in cell 1, and W,X,Y and Z in cells 3,5,7 and 9, respectively. Each edge cell can now contain what is in the two corners opposite it. Cell 2 can contain YZ, cell 4 can contain WZ, cell 6 can contain ABC...V plus Y, and cell 8 can contain ABC...V plus W. This is shown below.

ABC...V	YZ	W	22 2 1
ZW	X	ABC...V,W	2 1 23
Y	ABC...V,W	Z	1 23 1

This example shows the most unbalanced possible letter distribution which would achieve the theoretical maximum score. We can also achieve this score with balanced letter distributions. By placing N in the center and dividing the rest of the alphabet into close to equal chunks (A-G, H-M, O-T, U-Z), we can construct the following grid.

ABCDEFG	OPQRSTUVWXYZ	HIJKLM	7 12 6
HIJKLM,UVWXYZ	N	ABCDEFG,OPQRST	12 1 13
OPQRST	ABCDEFGHIJKLM	UVWXYZ	6 13 6

This example more clearly shows how edge words are constructed out of the pool of letters in the two opposite corners (plus any letters not used elsewhere in the grid, of which there are none in this example). We see the challenge of finding a quadrisogram for the corner cells such

that word pairs in each pair of adjacent corners is fertile enough to allow a long word in the opposite edge cell.

GETTING REAL

We might next inquire what happens to the theoretical best grid when we include some assumptions about real words. Our (25,25,26) grid would contain two edge pairs with 25 letters. However, given currently-known word sets, two words can contain at most 20 letters, and only one 20-letter solution is known: GUNPOWDERY BLACKSMITH. The other edge pair can then contain a maximum of 19 letters. As mentioned, a list of 19-letter bisograms has been compiled for this purpose. This makes our theoretical maximum grid at most (20,19,26) for a total of only 65--and it would have to be based on GUNPOWDERY BLACKSMITH. How the mighty have fallen! Next we see what happens to our theoretical maximum when we examine the checkerboard.

To keep the theoretical maximum score at 65, we would need to find either a 26-letter quintisogram (5x26)--i.e., a five-word pangram--or alternatively a 25-letter quadrisogram (5x25) with all words acceptable in W3. Do such beasts exist? Ross Eckler covered these questions in the February 1977 Word Ways. He reported no four-word set which uses 25 letters, and reported only Borgmann's PHLEGMS FYRD WUZ QVINT JACK-BOX as a five-word pangram, the latter three words of which are not acceptable in W3.

As reported in a previous article, a family of five-word pangrams does exist in W3: THUMB FROWZLY VEXING JACKS PDQ. Both the -ING and the -S can be moved around to create related forms, none of which much affects scoring potential. Given this example, our theoretical best grid remains at (20,19,26) for a total of 65. However, actually constructing such a grid is clearly not possible, as there is no way to place FROWZLY VEXING JACKS THUMB PDQ (or any of the related forms) into the checkerboard so as to allow GUNPOWDERY or BLACKSMITH in an edge cell. We can safely say that without some new discoveries, no grid can have 26 letters in the checkerboard and 20 letters in one of its bisograms. Therefore a score of 65 is not possible.

WORKING OUR WAY DOWN

We next check the possibility of scoring 64. This is achieved with either of two grids: (19,19,26) or (20,19,25). The first of these would have to use the above-mentioned pangram with two 19-letter bisograms. This possibility is examined below, showing the three possible placements of the five-word pangram in the checkerboard. Letters in brackets represent pools of available letters. Letters which can go in one of multiple cells are listed after a slash; others have only one possible location.

frowzly	[thumbjacks]	vexing
exhuming[btv]	pdq	[frowzlyjacks]
jack/s	overflying[wxz]	thumb/s

frowzly	[thumbjacks]	vexing
[vexingjacks]	pdq	[frowzlythumb]
thumb/s	overflying[wxz]	jack/s

frowzly	exhuming[vtb]	jack/s
[jacksvexing]	pdq	[frowzlythumb]
thumb/s	[frowzlyjack/s]	vexing

A given four-word set can be placed in the corners in three ways which are not equivalent by rotation or translation. The above grids show the most balanced forms of the pangram, in its three possible placements. Some sample edge words are filled in. Clearly, PDQ must go in the middle. While we can find long words for a few edge cells, others do not lend themselves as reaidly. We can also examine the list of known 19-letter bisograms. For a bisogram to be possible in this grid, it must not use the letters PDQ. Only two such 19-letter examples are known: BACKLIGHTS JURYWOMEN and THUMBSCREWING ALKOXY. Neither of these works with any arrangement of corner words, or the related forms. Thus, unless a new 26 or some new 19s are found, a (19,19,26) remains impossible.

The second potential way to score 64 would be a (20,19,25) grid using GUNPOWDERY BLACKSMITH and a 25-letter checkerboard. In examining this possibility, we start by noting that once these two words are used, the six extra letters, FJQVXZ, do not allow any two-letter words. So the center must contain only one letter. In order to reach our desired total of 25 letters for the checkerboard, we will therefore need to add a net of four of the extra letters into the corner words, two of which will be made from the letters of BLACKSMITH plus extra letters, and two of which will be made from the letters of GUNPOWDERY plus extra letters. A manual search of all possibilities shows that this cannot be done. One can add at most two extra letters to GUNPOWDERY (for example, FROWZLY EXPUGN), and at most one extra letter to BLACKSMITH (for example, KLATSCH ZIMB). This is not enough to allow 25 letters in the checkerboard. The methods for accomplishing these additions are further discussed below. Given this fact, short of discovering either a new 20-letter bisogram, or a way to add a net of four letters to the existing one which was missed, no grid can be of the form (20,19,25). We cannot score 64 --onwards and downwards!

THE 63-LETTER QUESTION

A score of 63 could conceivably be achieved by two grids, (20,19,24) and (19,19,25), having already ruled out (19,18,26) and (20,18,25). We first consider a (20,19,24 grid), which would again need to start with GUNPOWDERY BLACKSMITH, and add three extra letters (from FJQVXZ) in the corner squares. This turns out to be possible. A maximum of one letter can be added when making two words out of BLACKSMITH, accomplished in the following ways:

BLACKFISH QT adds FQ, removes M, net +1
KLATSCH ZIMB adds Z, removes no letters, net +1
SCHMALTZ FIB adds FZ, removes K, net +1
SCHMALTZ JIB adds JZ, removes K, net +1
SCHMALTZ KIF adds FZ, removes B, net +1

A maximum of two net letters can be added to GUNPOWDERY:

FROWZY EXPUGN adds FXZ. removes D, net +2
UPGROWED JYNX adds JX, removes no letters, net +2

Of these, FROWZY EXPUGN cannot be paired because all ways of adding net 1 to BLACKSMITH also use either F or Z. However, UPGROWED JYNX allows for four complementary pairs:

UPGROWED JYNX and BLACKFISH QT adds FJQX, removes M, net +3
UPGROWED JYNX and KLATSCH ZIMB adds JXZ, removes none. net +3
UPGROWED JYNX and SCHMALTZ FIB adds FJXZ, removes K, net +3
UPGROWED JYNX and SCHMALTZ KIF adds FJXZ, removes B, net +3

These are the four possible ways to add a net of 3 of FJVQXZ to our initial 20-letter bisogram. Placing one of the remaining letters in the center will bring the checkerboard total to 24. The challenge is to find a 19-letter bisogram for the remaining edge pair. The analysis for the first complementary pair is given below; similar arguments apply to the others. Placing Z in cell 5, the grid is:

blackfish	gunpowdery	qt
	z	
jynx	blacksmith	upgrowed

There are 10 letters that can appear in cell 4 (DEGOPQRTUW), 13 that can appear in cell 6 (ABCFHIJKLNSXY), and two (MV) that can appear in either cell, but not both simultaneously. The cell 6 letters can be arranged to form the nine-letter KNAVISHLY, but then it is impossible to find a ten-letter word from the letters in cell 4 (although PTERYGODUM misses by only one letter, the Y).

We next consider the possibility of a (19,19,25) grid. Of the known 19-letter bisograms, none leaves a word of five or more letters; thus the checkerboard must include a word of three or fewer letters for the center cell. This eliminates solutions to the five-letter Jotto Problem (4x25s with distribution 5,5,5,5), and 4x25s in which the shortest word is four letters. We consider other possible letter distributions by gradually decreasing the length of the center word.

Three-Letter Center Word

In order to place a three-letter word in the center, we must find a three-letter word left over by two different 19-letter bisograms. The only such examples are:

PDQ BACKLIGHTS JURYWOMEN and THUMBSCREWING ALKOXY
VEX THROWBACKS JUMPINGLY and THUMPINGLY BACKSWORD
FEZ THROWBACKS JUMPINGLY and THUMPINGLY BACKSWORD

Each possibility allows one placement in a grid. Again, we illustrate with a sample argument. The first grid is

	backlights	
thumbscrewing	pdq	alkoxy
	jurywomen	

There are 7 letters that can appear in cell 1 (FJOVXYZ), 11 in cell 3 (EFJMNORUVWZ), 7 in cell 7 (AFKLVXZ) and 10 in cell 9 (BCFGHISTVZ), but no letter can appear in more than one cell. Working on the cells with fewest letters first, one places JOY in cell 1 and FLAX in cell 7; then BIGHTS is the longest word for cell 9 (F is no longer available), and one is faced with the impossible task of aswsembling a 9-letter word out of EMNORUVWZ. Using similar arguments for the other cases, one can show that no (19,19,25) is possible with a three-letter word in the center.

Two-Letter Center Word

In order to place a two-letter word in the center, it again has to be left over by two different 19-letter bisograms. The words which allow this are JG, JP, VD and QT. These are contained in the following 5x25s.

JP BACKSWORDING ZYTHUM FLEX Q (or V)
JP VETCHWORM SQUDGY BLANK FIX
JP VETCHWORM SQUDGY FLANK BIZ
JP BARUZHKY FOXING VELDTS CWM
JP BOXINGLY KVUTZAHS DERF CWM
QT FROWZLY VEXING BUMPH JACKS
QT HUMPBACKS FROWZY JINGLE VD
QT HUMPBACKS FROWZY DELVING X (or J)
QT HUMPBACKS FROWZY JINGLED X (or V)
VD HUMPBACKS FROWZY JINGLE QT
VD THROWBACKS JUMPINGLY FEZ Q (or X)

Given these, we can immediately see that JG is not a possible center word. None of the JP examples support known 2x19s, or provide a realistic possibility of any 2x19s. However, the first QT example provides a tantalizing near-miss.

frowzly	humpbacks	vexing
	qt	
jack/s	overflying[wzxd]	bumph
	oxywelding/s[frvz]	

Two placements allow the 19-letter bisogram HUMPBACKS OVERFLYING and HUMPBACK/S OXYWELDING. A score of 63 could be achieved by finding a 2x19 from the letters ACDEGIJKNSVX in cell 4 and BDFHLM-OPRSUWYZ in cell 6, or BDEGHIMNPSUVX in cell 4 and ACDFJKLORSWYZ in cell 6. The vowel distribution is good, but the consonants are unlikely.

One-Letter Center Word

Having eliminated all other possible (19,19,25) grids, we examine those with one letter in the center. In such a grid, the checkerboard must contain a 4x24 quadrisogram in the corners. We examine the list of known 4x24s, some of which have already been used as 5x25s with placements involving two-letter center words. Most are in the HUMPBACK family, one is a derivative of a 2x19, and one is unrelated to previous discoveries.

BACKSWORDING ZYTHUM FLEX JP (Q or V)
THROWBACK/S JUMPINGLY FEZ VD (Q or X)
HUMPBACK/S FROWZY VELDT JINX (G or Q)
HUMPBACK/S FROWZLY VEXT DING (J or Q)
HUMPBACK/S FROWZLY VEXT JING (D or Q)
HUMPBACK/S FROWZLY VEST DJIN (G or Q)
HUMPBACK/S FROWZY DELVING QT (J or X)
HUMPBACK/S FROWZY JINGLED QT (V or X)
PROXYING KVUTZAH/S FJELD CWM (B or Q)

While some of these again allow HUMPBACKS OXYWELDING or HUMPBACKS OVERFLYING as described above, none allows a companion pair. So in order to have a (19,19,25) grid with a one-letter center, we would need a new 2x19, or a new 4x24, but with currently-known word sets none is possible.

AND FINALLY...

Four forms of grids can score 62: (18,18,26), (19,18,25), (19,19,24) and (20,19,23). Since our lists of 5x25s, 5x26s and 2x20s is very limited (to one example each in the latter two instances), we will not cover these cases. We do, however, have a promising list of 2x19s which we can attempt to combine with a 24-letter checkerboard. While still challenging, a 5x24 is still within the realm of possibility. Can we achieve a score of 62? Indeed, the contest winner, Lambert Bright of Lincoln, Nebraska, used a (19,19,24) grid to score 62, as did the tying grid found by my brother and me.

proxying	buckwash	veldt	(Bright)
stumbled	j	zincography	
whack	providently	bumfs	
thump	overflying	backs	(Chaikin)
backsword	q	thumpingly	
vexingly	humpbacks	fjord	

In the first grid, the interlocking of the 2x19s is very subtle: the corner words are not derived from components of the edge cells. In the second grid, the components break apart and reassemble in the corners with surprising ease. From BACKSWORD, BACKS and FJORD pick up a missing FJ while dropping only a W, while THUMP and VEXINGLY pick up a missing EVX while dropping nothing, for a net addition of four letters. Note that cell 2 can also contain OXYWELDING.

Some of the grids above may lend themselves to other scores of 62, but having shown that this is the maximum possible score, we have achieved our goal and will stop here, awaiting more contributions to the N-isogram reference lists. We can rest peacefully, having stuffed the isogrid to the limit.

CHALLENGES

Given what has been shown above, any of the following challenges would need to be met in order to top a score of 62:

- a new 10- or 11-letter isogram in some of the letter stocks given above
- a new 2x19, specifically one which would fit with a 5x25, or with the known 2x20
- a new 2x20 which would fit with a 2x19 and a 5x24
- a new 5x26 which would fit with two 2x19s, or a 2x19 and a 2x18
- a new 5x25 or 4x2k4 which would fit with two new or existing 2x19s

Until one of the above challenges is met, the isogrid is stuffed at 62--and so am I. Have at it, Word Ways readers!

VERBATIM RESURRECTUS

Word Ways is delighted to report that, phoenix-like, Laurence Urdang's magazine Verbatim has risen from its ashes (see Word Ways August 1997) thanks to a financial angel, Dr. Warren Gilson. Edited by Erin McKean in Chicago, it is available for $25 per year; send money to 4907 N Washtenaw Ave, Chicago 60625. The first issue (Autumn 1998) contains a clutch of interesting articles including a discussion of chemical element names, a progress report on the *Dictionary of American Regional English* (Volume 4 by 2002?) with a 1968 eyewitness account of the data-collection process, an extended book review of the best-seller *The Professor and the Madman*, and the revision of *The F-Word*.