HOW MANY ALPHOMES ARE THERE?

REX GOOCH
Welwyn, Herts, England

An alphome consists of the letters of a word arranged in alphabetic order. In "The Commonest Alphomes" in this issue, I presented the most common alphomes for many word-lengths. Here I look at alphomes en masse, but only dealing with those found in well-known dictionaries.

If we have a word with \( n \) letters, the number of possible words that can be made, from AAA..A to ZZZ..Z, is \( 26^n \). At 10 letters, the number of possible words could fill 23 billion Bibles with every word different!

The number of alphomes for single-letter words is 26, the same as the number of words. To find the number of two-letter alphomes, we multiply 26 by 27/2, giving 351, as compared with 676 possible words. To find the number of three-letter alphomes, we multiply 351 by 28/3. For four-letter alphomes the multiplicative factor becomes 29/4, and so on. Observe that, as we add one to the word length, the number of possible words includes by a factor of 26, whereas the number of possible alphomes increases by a factor which is never greater than 13.5 and steadily decreases towards one. Thus at each successive step there are ever more words per alphome, so anagramming should be increasingly easy. Indeed, on this model, the number of words per alphome (the average number of "anagrams") increases from one for words of length one, through 420 for words of length six, to 768,970 for words of length ten!

This is completely misleading, because, as shown in Graph 1 of my article "Statistics of Word Neighbours" in the November 1997 Word Ways, the fraction of possible words that actually exist decreases sharply with each additional letter (beyond two) in the word; indeed, even the absolute number of words decreases after length ten. In the present article, Graph 1 shows that the actual number of dictionary words per alphome increases from one for words of length one, through a peak of 2.5 for words of length four, before falling away sharply to values of 1.1 or below for words of length ten or more. This implies that words of length ten or more do not often have anagrams, and rarely have two or more anagrams (see "The Commonest Alphomes" for examples).

Graph 1 shows length six to be an intermediate example, with an average of 1.9 words per alphome. The distribution about this average is all-important, and is shown in Graph 2. 60 per cent of alphomes corres-

Copyright Rex Gooch 1999
GRAPH 1
Words per Alphome

GRAPH 2
Percentage of alphomes of length 6 having 1, 2, 3... words
pond to only one word, but nearly 20 per cent have two words, nearly 10 per cent have three words, and so on. The result is perhaps more significant than one might guess. Graph 3 shows that 70 per cent of six-letter dictionary words are anagrammable; indeed, it shows that a reasonable percentage of words are anagrammable up to and beyond ten-letter ones. The small variations beyond length 20 are not very interesting, commonly being explained by the interchange of portions of a word (like HEPATO and BRONCHO), as explained in "The Commonest Alphomes".

GRAPH 3

Percentage of Words which are Anagrammable