SOME PROPERTIES OF ONE, TWO, THREE...

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In this article we explore some properties of the number names: the results of looking at the names in order, or looking at the whole string ONE, TWO, THREE, ...

SOME BASIC PROPERTIES

As one proceeds through the string ONE TWO THREE..., the order in which the letters of the alphabet occur is ONETWHRFUIVSXGLYDAMBQPC, with J, R, and Z never appearing. Eighteen of these 23 letters have appeared by the time we get to ONE THOUSAND, but it takes a long time, comparatively, to get to the last five (MILLION, BILLION, QUADRILLION, SEPTILLION, OCTILLION).

The smallest number whose name contains (at least) k different letters, for k = 1 to 20, is:

3	ONE (also first 1 and 2)
4	THREe
5	EIGHT
6	THIRtEeN
7	FOURTEeN
9	TWENtY-FOUR (also first 8)
10	SEVeNTY-FOUR
11	ONE HUnDRed TWeLVe
13	ONE HUnDRed TWentY-FIVe (also first 12)
14	ONE HUnDRed SIXTY-FiVe
15	TWO HUNDREd SIXtY-FiVe
16	ONE THoUSAnD tWo hundRed sIXtY-FiVe
17	TWO tHoUSAND FIVE hundRed siXtY-eiGht
18	TWELVe tHOUSAND FouR hundred sIXtY-eiGht
19	ONE MILlion TWo tHoUSAnD FiVe hundRed siXtY-eiGht
20	ONE BILlion one Million TWo tHoUSAnD FiVe hundRed siXtY-eiGht

The final three letters are added by incorporating QUADRILLION, SEPTILLION and OCTILLION into the last number name.

Sometimes we wish to examine the whole string from ONE to some number n. A fundamental question that then arises is: how many total letters are there? Of course we can just count them up one number at a time, but we seek a more efficient method. This can be done in general, but for brevity we just give the result for n equal to a thousand minus 1, a million minus 1, and so on.

The number of letters in the words for 1 to 10^n-1 is $10^{n-3}t(n)$, where
$t(3) = 18440$
$t(n) = t(n-3) + t(3) + 999$(number of letters in the word for 10^{n-3})
for $n>3$ and a multiple of 3

For example, take n=6. The number of letters in the words 1...999999 is:

1000 times t(6-3) + t(3) + 999(number of letters in THOUSAND) =
1000(18440 + 18440 + 999(8)) = 44872000

which can also be verified by brute-force counting. This tells us that if we choose a number at random from this range it will, on average, use about 45 letters. Between one and one billion it's about 70, and between one and one trillion, about 96.

A somewhat different question is: how big a set of letter tiles is needed to spell any number in the range from 1 to n? For example, 37 tiles with distribution (0 0 0 2 4 3 2 2 2 0 0 1 0 4 2 0 0 2 2 3 1 2 2 2 1 0) are sufficient for n=100. The answers for n equal to 10, 100, 1000, etc. are: 16, 37, 59, 82 101, 125, 148; each additional power of 10 increases the count by about 23.

Many of these same questions can be explored using the string ZERO, ONE, TWO, etc. One odd discovery is the fact that the number of letters in ZERO, ONE, TWO ... SEVENTY-NINE is equal to the number of the beast (666). No initial segments of ONE TWO THREE... have this property.

GEOMETRIC PACKING FUN

For some values of n, the words ONE, TWO, THREE ... (n) can be packed into a rectangle one of whose sides is n. This is a notable property, as there are, for example, only 14 values of n less than a million for which this is true:

1	1x3	2595	2595x25
2	2x3	3870	3870x27
8	8x4	5568	5568x28
9	9x4	32592	32592x33
31	31x7	55019	55019x34
283	283x15	264740	264740x41
786	786x18	426067	426067x43

Actually, this list just shows which ones *might* be able to be rectangle-packed, i.e., those for which the number of cells in the rectangle is equal to the number of letters in the string. One of these could, in theory, be unpackable; but we (somewhat boldly) conjecture this is never the case, based on being able to pack the first four, plus the fact that larger ones should be easier to pack.

For n=9, we note that 9x4 is also 6x6, so this motivates the question: what sequences of ONE through (n) can be packed into a square? There are many of these for n less than ten thousand (first number is n, second number is the size of the square):

9	6	332	72	1683	195	4365	345	8243	487
16	9	353	75	2600	255	4585	355	8605	499
21	11	389	80	2945	275	6235	419	8605	499
56	21	494	92	3026	279	6683	435	9341	521
107	31	765	117	4008	330	7507	463	9742	533

Again, these are only values for which a square packing *might* be possible. The first packing is possible, as shown below:

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o T f o u r
n W F I V E
e O s i x N
T H R E E I
s e v e n N
E I G H T E
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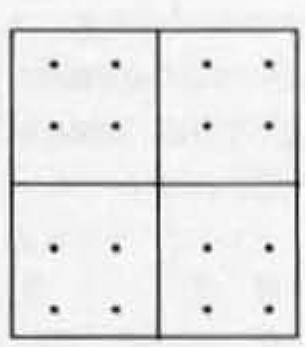
The second one on the list (ONE through SIXTEEN fitting in a 9x9 square) ranks as perhaps the most amazing discovery made in this investigation, because both n (16) and the number of points in the square (81) are fourth powers! This means that, as shown in the figure below, these both fill (not pack) four-dimensional hypercubes:



(a) 16 points fills a 2x2x2x2 hypercube

e	S	t	F	n	i	n	e	t
i	E	h	O	S	I	X	e	e
g	V	r	U	o	n	e	l	n
h	E	e	R	T	W	O	e	T
t	N	e	f	i	v	e	v	W
S	I	X	T	E	E	N	e	E
f	i	f	t	e	e	n	n	L
T	H	I	R	T	E	E	N	V
f	o	u	r	t	e	e	n	E

(b) ONE...SIXTEEN fills a 3x3x3x3 hypercube

The diagram also shows how to pack ONE ... SIXTEEN into a 9x9 square, thus proving that the second item on the list above can be packed. Packing the larger squares is left as a puzzle for the reader.

A probabilistic calculation suggests that the number 16 is unique--there are *probably* no other values of n with this doubly-hypercube-filling property. We checked all numbers up to one hundred million and did not find another example.