PACKING THE CARDINALS

A. ROSS ECKLER
Morristown, New Jersey

In a companion article, Michael Keith discusses the problem of packing the cardinals ONE, TWO, THREE, ... N into various square or rectangular grids of minimum size. This article extends some of his ideas, showing that such grids come in several varieties.

Grids of size 1xN can always be packed efficiently, and are not discussed further; grids of size 2xn are almost as trivial to fill. We consider only those values of N for which both sides of the minimum rectangle are three or larger.For grids containing the first N cardinals:

4	3x5	10	3x13	17	9x10
7	3x9	11	5x9	18	7x14
8	4x8	12	3x17	20	7x16, 8x14
9	6x6, 4x9	16	9x9	21	11x11

One can distinguish three varieties of grids: cross-grids containing both vertical and horizontal cardinals, linear grids in which all cardinals are horizontal, and ordered grids in which adjacent cardinals are always touching (at more than just a corner). We present linear or ordered grids whenever possible. Linear grids are more likely in elongated rectangles, and ordered grids in rectangles approaching a square shape. For N= 8 and 12, it was possible to find linear ordered grids, the only non-trivial examples of these known (excluding the examples for N=1,2). Note that square grids are possible for N= 9,16,21.

```
7 O N E S E V E N s
  T W O T H R E E i
  F O U R F I V E x
  (ordered)
```

```
8 O N E E I G H T
  T W O S E V E N
  T H R E E S I X
  F O U R F I V E
  (linear, ordered)
```

```
9 E I G H T n
  S E V E N i
  s F I V E n
  i F O U R e
  x T H R E E
  O N E T W O
  (ordered)
```

9 add NINE vertically down the right edge of 8 (ordered)

```
10 t O N E T H R E E F O U R
   w S E V E N F I V E S I X
   o E I G H T N I N E T E N
```

```
11 E L E V E N T E N
   E I G H T N I N E
   S E V E N S I X o
   F O U R F I V E n
   T H R E E T W O e
   (ordered)
```

```
12 T W E L V E T H R E E T W O O N E
   T E N E L E V E N F O U R F I V E
   N I N E E I G H T S E V E N S I X
   (linear, ordered)
```

```
16 T W E L V E T E N
   t f E L E V E N n
   h o T H R E E t i
   i u f F O U R w n
   r r i s O N E o e
   t t v i E I G H T
   e e e x S E V E N
   e e S I X T E E N
   n n F I F T E E N
   (ordered)
```

```
18 S E V E N T E E N T H R E E
   F O U R F I V E E L E V E N
   T H I R T E E N T W E L V E
   S E V E N E I G H T N I N E
   F O U R T E E N O N E T W O
   S I X T E E N F I F T E E N
   E I G H T E E N S I X T E N
   (linear)
```

17 add SEVENTEEN vertically down right edge of 16 (ordered)

```
21 T W E N T Y O N E s t
   S E V E N T E E N e h
   E I G H T E E N t v r
   N I N E T E E N e e e
   T H I R T E E N n n e
   F O U R T E E N S I X
   F I F T E E N t f f n
   S I X T E E N h o i i
   T W E L V E t r u v n
   E L E V E N w e r e e
   T W E N T Y o e O N E
```

There are too many long cardinals in the N=21 grid to make either an ordered or a linear grid possible. One saves the short cardinals (ONE, TWO, ... TEN) for use in the lower right corner after as many of the long cardinals have been accommodated as possible.

Note that there is one grid that is impossible to pack: the 3x5 grid with ONE, TWO, THREE, FOUR. It is conjectured that all other grids can be minimally packed, although not necessarily in a linear or ordered format. How large can either a linear or an ordered grid be? (For the latter, I'd guess N=17.)