EDITH PLAYS WORD TREBLECROSS

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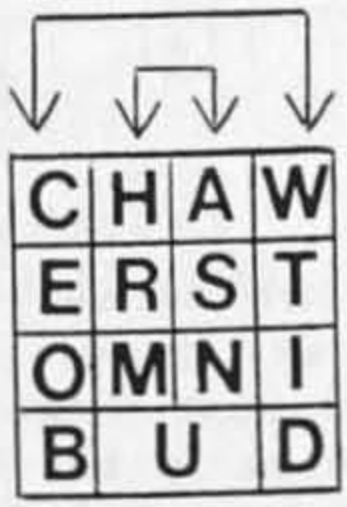
Ordinary Treblecross is played by two persons who alternately place the same symbol, a cross, on a 1-by-n strip of squares with the winner being the first player to complete a row of three consecutive crosses. In his very readable book *Fair Game* (Comap, 1989) Richard K. Guy observes "that it's stupid to play next, or next-but-one, to a cross that's already been made: your opponent could complete three consecutive crosses immediately." Guy continues his study of the game by showing that Treblecross is equivalent to the combinatorial game Nim. Readers of Word Ways may recall the use of Nim analysis in the article "Wordnim and Grundyword" in the August 1993 issue.

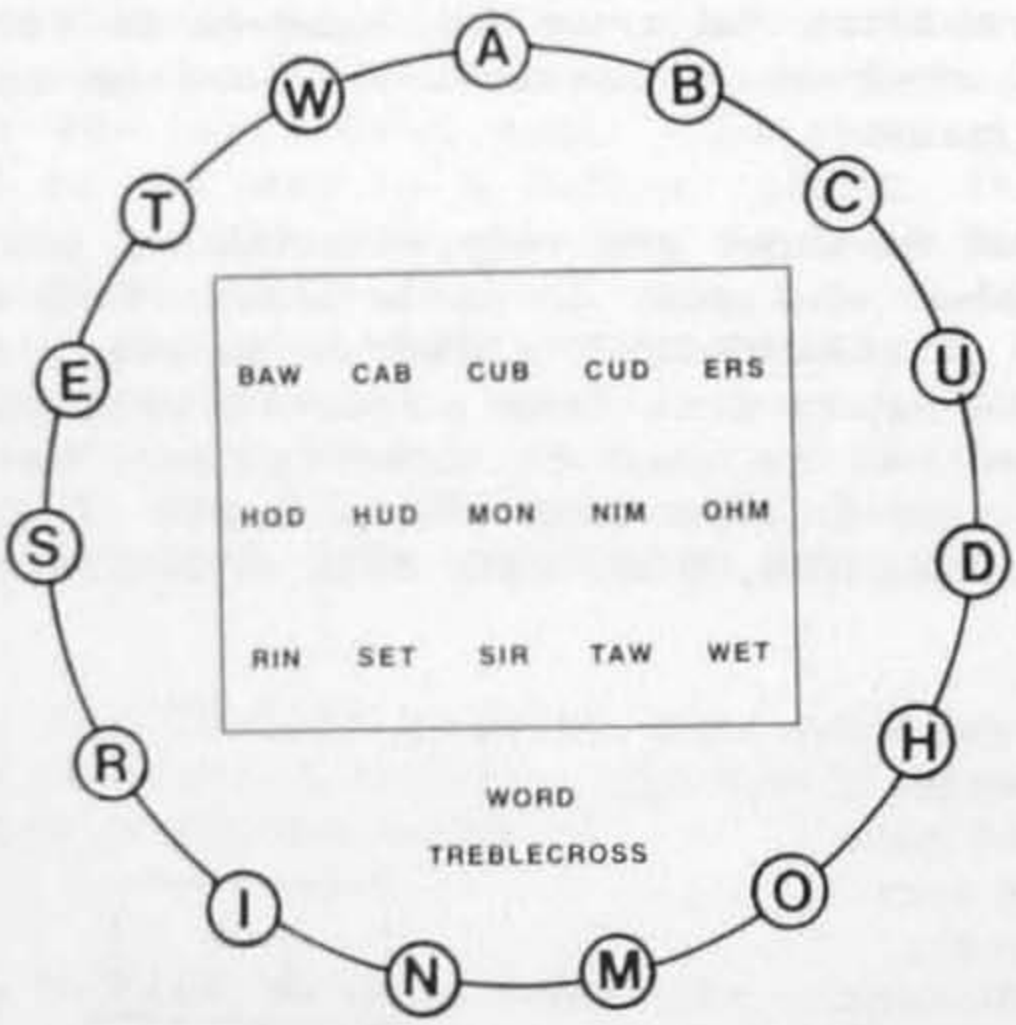
We recently have developed what we think are very entertaining word versions of Treblecross. The simplest and most direct is Linear Treblecross where we write the letters of SCRUBWOMAN EDITH on fifteen tiles called scrubs. The two players alternately draw from a face-up bone pile of scrubs to form a pool of letters, to be used by either player, until one of them wins by first being able to form from the pool one of the words BUD, BUM, CAT, DIN, DUN, HER, HEW, MOB, ORE, ROM, SAT, SIN or SIT.

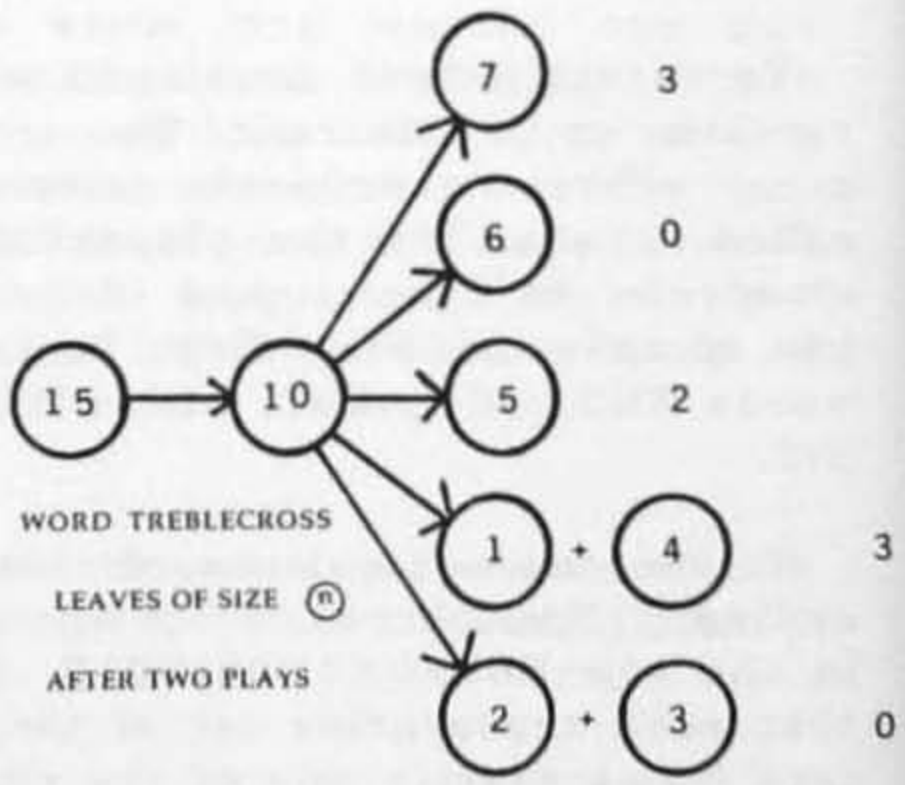
To see the equivalence of this game to ordinary Treblecross, write the letters in the row WHEROMBUDNISTAC, and note that each consecutive set of three letters forms exactly one of the thirteen allowed words. A little thought will convince the reader that the first player, F, has a forced win over the second player, S, in this game. F simply takes initially the center letter U and answers any safe reply by player S by drawing the scrub that is the symmetrical mate of S's play. For example, if S initially playes A, F replies with H, and so on. Sooner or later, F must win. (Be careful to always check for instant wins in case S blunders!) How do we remember the symmetries? We use the mnemonic word rectangle at the right.



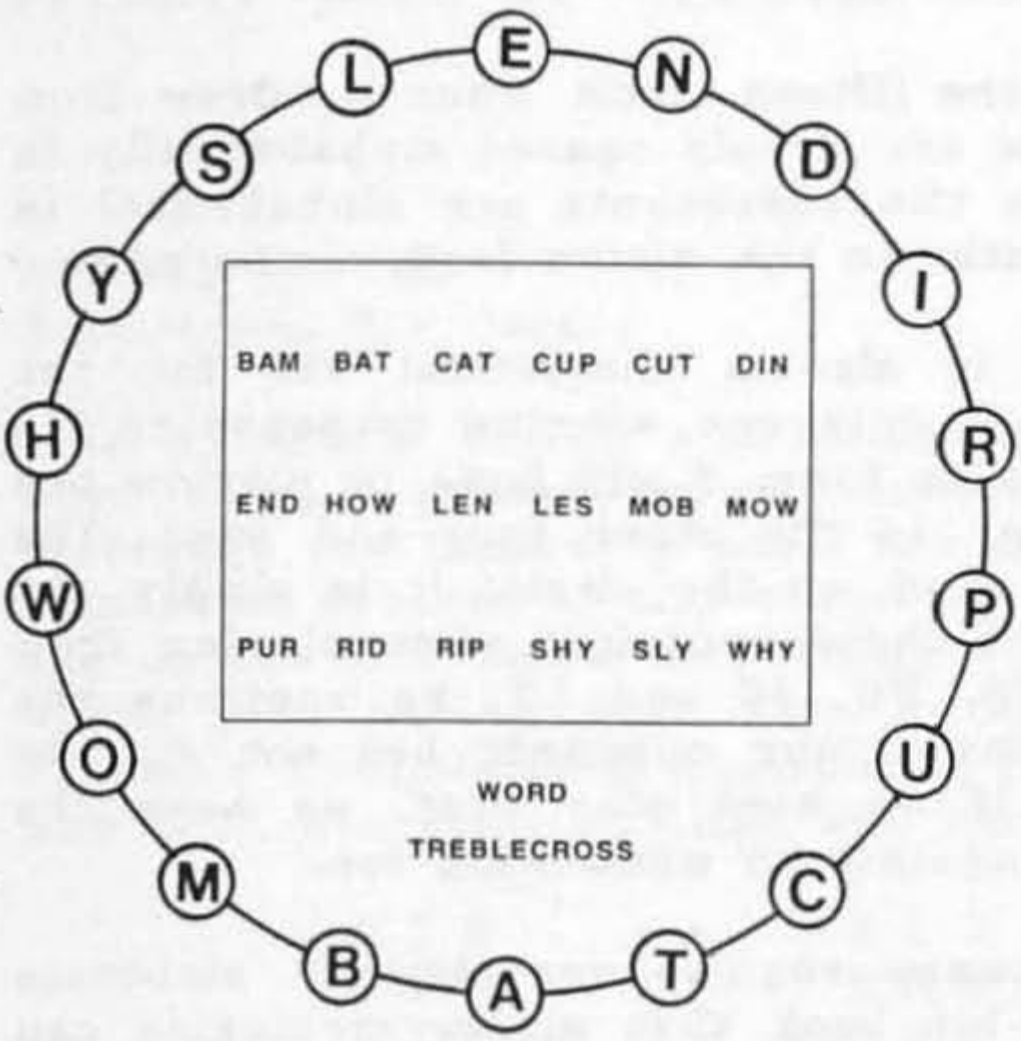
SYMMETRIES IN LINEAR TREBLECROSS

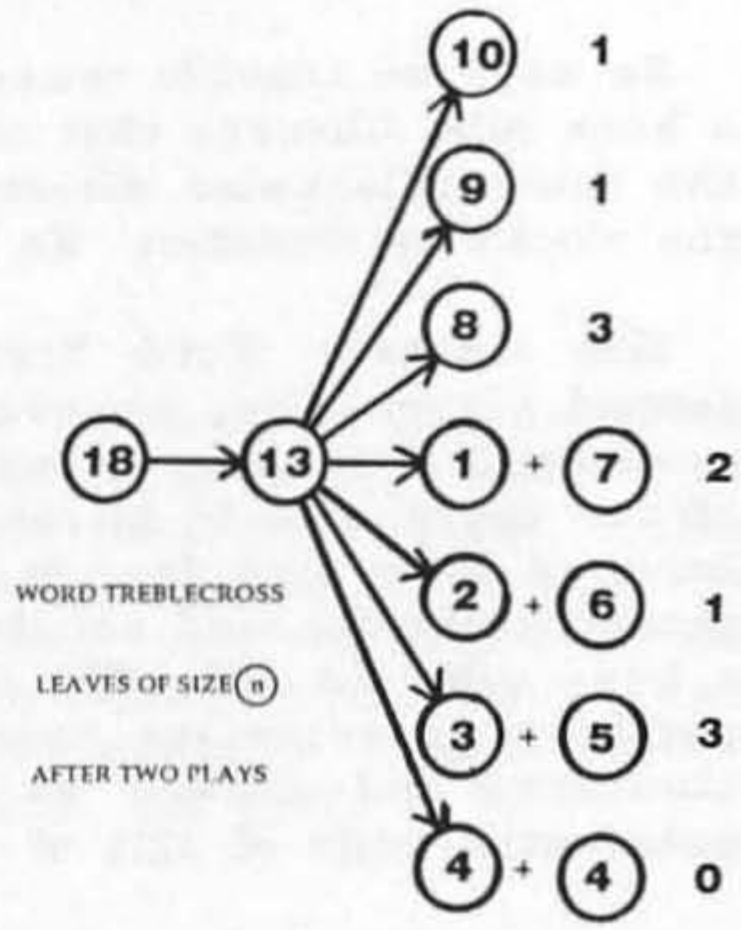
No use of Nim was necessary for Linear Treblecross (the symmetry technique is called Tweedledee-Tweedledum) but we will certainly need it for our two versions of the games we call Word Treblecross. The fifteen-word game again uses the letters of SCRUBWOMAN EDITH, now arranged around a certain circle, while the eighteen-word version uses the letters PLY SCRUBWOMAN EDITH in its cyclical arrangement. In both, every set of three consecutive letters forms one of the allowed words shown in the lists inside the respective circles. Any unfamiliar words can be found in the Merriam-Webster New International, Second Edition.We prefer to play either version by suppressing the circles and showing only the word lists, drawing again from a face-up bone pile, until one player first is able to form an allowed word from the common pool of drawn scrubs. Played in this manner, Word Treblecross is far from easy. We also have misère forms of the games where the first player to be able to form a word loses. That is, all currently drawn scrubs are regarded as in play and if any listed word is possible among any set of three of these, the player on move loses.





In the normal version of fifteen-word Word Treblecross, there are, after any draw, certain safe "leaves", i.e., certain scrubs that can be played next without immediately losing the game. These are shown for two plays in the tree diagrams where the circled numbers are the size of the leaves. Associated with these leaves are Nim values called nimbers that allow us to compute the leaves we want to give our opponent. Briefly, Nim strategy says that a non-zero nimber can be changed by at least one move to a 0 nimber and a nimber of 0 must <u>always</u> be changed to a non-zero nimber. Hence we strive to leave a 0 for our opponent and, if we can do so, he must leave a non-zero for us. This is repeated until our foe has the ultimate 0, that is, no move at all and loses.





It often occurs that a play will separate our scrub circle into several parts. To compute the final nimber of this position, we must use the nim-sum addition table to combine the individual nimbers of the parts. For instance, in the tree diagram for the eighteen game, a leave of thirteen may be separated by a draw into new leaves of one and seven. The nimbers of these two leaves are 1 and 3, which nim-sum to 2, the final nimber of the position.

THE NIMBER ADDITION TABLE

⊕	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

SIZE (n)	0	1	2	3	4	5	6	7	8	9	10	13	15	18
NIMBER	0	1	1	1	2	2	0	3	3	1	1	4	0	0

THE RELEVANT NIMBERS
USED IN WORD TREBLECROSS

Here is how to play expertly in the fifteen game. We notice that fifteen has nimber 0 so we would prefer to have our opponent start first. No matter which scrub he takes, say A, he leaves ten safe scrubs, here the string ESRINMOHDU. Ten has nimber 1 and we note in the fifteen tree that we could give our opponent a 0 nimber in several ways. We could leave six by drawing either D or S, or we could leave two plus three by drawing either N or M. If we choose N, for example, our opponent can only play safely in the set ES or the set HDU. Whichever he selects, we play in the other set and win. When we must

be first to play we still have odds of 3:2 of winning against a naive player who does not know how to leave an 0.

We have no trouble remembering the fifteen circle when we draw from a bone pile. Observe that the vowels are evenly spaced alphabetically in the counterclockwise direction while the consonants are alphabetical in the clockwise direction. We return later to the misère form of the game.

Size eighteen Word Treblecross is also a theoretical win for the second player. Now, however, there is only one winning response to F's opening; S must play to leave four plus four. F will have to play in one of the fours and S mirrors his play in the other four and wins. The leave of four plus four is easy to find on the circle; it is simply the diametrically opposed scrub. Memorize these opposites when playing from a bone pile: EA, NB, MD, IO, RW, PH, YU, SC and LT. We continue the matching on succesive rounds of play if our opponent has not already blundered and allowed us to win. If we must play first, we have the substantial odds of 12:1 of winning against an unknowing foe.

Many misère forms of nim-like games require surprisingly elaborate schemes to win. R.K. Guy notes in his book that misère strategies can not ordinarily be developed along nim-sum lines. However, for eighteen Word Treblecrooss, we have an easy strategy for the misère game. The second player wins by blindly playing in the list of opposites given above. First will always lose.

For the fifteen misère game, the play is much more complicated. It is the first player that now has the forced win and he starts by drawing any scrub. After S's response, it will always be possible for F to make a play that separates the circle into one of the three sets of leaves (two, three, seven), (zero, four, eight) or (one, four, seven). Play can be tricky as the game continues, but F can prevail. It is best to practice this misère game on the fifteen circle to gain facility.

We close with a challenge. Perhaps some energetic reader will be able to arrange the entire alphabet into a circle so that every set of three consecutive letters forms a reasonable word or abbreviation. If so, they will have constructed twenty-six Word Treblecross and will find on analysis that the second player wins either the normal or misère forms of the game.