A NEW KIND OF TRANSPOSAL SQUARE

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In my article "Single and Double Transposal Squares" in the May 1980 Word Ways, I generalized the concept of word square, allowing one to rearrange the letters in each row and column to form a word. Double transposal squares of size three are ridiculously easy to construct. The one on the left below is perhaps the commonest one, consisting of six words each having more than a thousand occurrences in Kucera and Francis's Computational Analysis of Present-Day American English (1967): man 1207, him 2619, its 1858, was 9816, who 2252, not 4609.

М	Ι	Н	100	A	Ι	0
A	S	W		H	Ν	S
N	Т	0		Т	W	Μ

The letters of a double transposal square of size 3 can be rearranged to form an array with a different set of properties. In the 3-by-3 grid shown at the right, the same six words can be read off in a different way: the letters of each word appear once in each row and once in each column.

This arrangement can be generalized to larger arrays. For instance, in a 4-by-4 array, there are 24 different ways in which one can select sets of four letters, one from each row and one from each column. Can an array be constructed in which all 24 letter sets are rearrangable to words? This can be done; the words from the array below can all be found in the second edition of the unabridged Merriam-Webster:

Y	Н	В	K	cyst	hunt	hind	etch
Е	S	N	L	drys	rusk	sick	herd
Ι	A	R	т	lory	balu	boil	kore
U	0	С	D	lacy	kuan	kino	bade
				Andy	hur1	bids	cake
				tony	bust	lich	bote

It is obviously impossible to construct a 5-by-5 array with 120 different words out of an array of 25 different letters. To have any hope of success, one must allow letter repetition. But here one must impose an additional restriction in order to avoid trivialities such as arrays consisting of the same 5-letter word repeated five times. The restriction is that no two of the 120 5-letter sets drawn from the rows and columns can be identical. It is a difficult mathematical problem to ascertain all the conditions guaranteeing that duplicates cannot be formed. (One such condition, for example, is that one cannot simultane-

ously place two As in columns a and b of one row, and two Es in columns a and b of a second row.)

Mike Keith programmed a computer to evaluate 5-by-5 arrays, checking which of the 120 5-letter sets in an array could be rearranged to form Merriam-Webster words. Not only did his program check that all letter sets were different, but it also evaluated neighboring arrays (those nearly the same as the original) to see whether a larger number of Websterian words could be found. Using techniques to move to everbetter arrays (climbing toward the summit of a mathematical "hill"), he discovered that one can frequently find arrays with 110 to 115 Websterian words--but a complete Websterian solution appears elusive.

Here are a couple of his best solutions, with 114 and 115 Websterian words, respectively:

	ELURL	RRRRL	
	ENELS	ACIMS	
	ARITS	AGETO	
	AMUST	ANUSA	
	ABODA	ABODE	
12345	anise scree	12354 tined cedar	12435 atune truce
12453	dunes duroc	12534 teton actor	12543 noses scroo
13245	erase girse	13254 deter argid	13425 ureal grume
12452	Druse drugs	13524 lerot morga	13542 roses gross
14235	AEEMT inert	14253 deems nidor	14325 melia EEMNR
14352	deism rends	14523 moles moron	14532 smote snort
15234	tebet ribat	15243 BEESS Boris	15324 blite amber
15342	bessi Serbs	15423 blues rumbo	15432 tubes burst
21345	aisle easer	21354 tiled ardea	21435 Aleut urate
21453	duels douar	21534 lotte aorta	21543 loses roosa
23145	urase gears	23154 trued garad	23415 urare argue
23451	lured dugal	23514 roter agora	23541 loser goals
24135	autem antre	24153 mused adorn	24315 aimer ranee
24351	limed laden	24513 mores aroon	24531 motel talon
25134	butte rabat	25143 buses boars	25314 tribe abear
25341	belis bales	25413 rebus burao	25431 bluet tubal
31245	asale arise	31254 dealt Daira	31425 alula AEMRU
31452	lauds sudra	31524 allot aroma	31542 lasso soars
32145	sauna cares	32154 daunt darac	32415 anura Eruca

32451 dunal ducal 34125 amula namer 34251 medal ladin 35124 tubal Abram 35241 bales bails 41235 alate irate 41352 dials reads 42135 AANTU trace 42351 ladin decal 43125 aural gamer 32514 trona caroa 34152 adsum darns 34512 roams arson 35142 ABSSU brass 35412 bursa bursa 41253 deals radio 41523 salol AMOOR 42153 Sudan cardo 42513 arson coroa 43152 sudra drags

32541 loans coals 34215 marae reina 34521 molal monal 35214 bater baria 35421 bulla album 41325 alila ameer 41532 altos roast 42315 arian ACEER 42531 talon octal 43215 arear regia

43251	alder algid	43512 roars sargo	43521 loral gloam
45123	albus broma	45132 tubas brats	45213 bares baroi
45231	table balti	45312 baris bares	45321 balli amble
51234	AELTT tiara	51243 sales orias	51324 aillt marae
51342	sails arses	51423 sulla amour	51432 talus sutra
52134	taunt carat	52143 Susan Oscar	52314 train areca
52341	snail scale	52413 ANRSU ACORU	52431 altun claut
53124	ultra grama	53142 sarus grass	53214 rater agria
53241	laser sigla	53412 surra sugar	53421 lural algum
54123	mauls manor	54132 tamus rants	54213 mares arion
54231	metal Latin	54312 maris earns	54321 milla leman

Can either word set be completed using words outside of Merriam-Webster? I suggest the following from the Times Index-Gazetteer and the Oxford English Dictionary:

AELTT attle, 'mining rubbish' ANRSU arsun, var of arson 'saddle-bow' AEEMT teame, var of 'team' (v2) BEESS Sebes, Rumania ABSSU Bussa, Nigeria or Ethiopia AANTU Nauta, Peru

EEMNR Merne Merna, Australia AEMRU Col de Maure, France AMOOR Moora, West Australia ACEER Cerea, Italy ACORU Coura, Portugal

In the January 1999 and April 1999 issues of Wordsworth, Ted Clarke discusses a similar problem entitled "Magic Word Squares". There he constructs a 6-by-6 array with all 26 letters of the alphabet represented in the 36 spaces. A complete set of 720 words is, of course, impossible to find; without spending much time on the problem, he located eleven 6-letter words in his array.