WORD KAYLES AND DAWSONWORD

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In "Wordnim and Grundyword" in the Aug 1993 Word Ways, and "Edith Plays Word Treblecross" in the Aug 1999 issue, Jeremiah Farrell, David Wright and Christopher Mihelich show how various nim-like games can be cast in the form of word games. This article casts two more such games in a similiar mould.

In the Farrell games, a single letter (or, in one of the games, an ampersand) is put on each of the "scrubs". A disadvantage of this is that the games are then hamstrung by the limitation of the size of the alphabet. (Why not "hamstringed"? The image is of doing something to the hamstring, not of stringing some ham.) The games described here use entire words, and this gives the games greater scope.

Kayles was described by Henry Ernest Dudeney in *The Canterbury Puzzles* (Thomas Nelson 1927), where Problem 73 relates to the game. Some skittles are set up in a line, and the players in turn throw a ball at them. The ball can knock down either one skittle or two that are next to each other. The object of the game is to knock the last one down.

Word kayles uses the successive frags in an ana-gram-mar chain. Each frag is required to be a word. The stockpile of allowable words consists of the individual frags and the words formed from any two successive frags. Each player may remove either the single frag or the two successive frags which make up one of the allowable words.

The strategy of kayles was described by Richard Guy and Cedric Smith in 1956. As with all the games mentioned so far, the value of a position is a nimber, and the good moves are those that leave the opponent a position where the nimbers nim-sum to zero. Farrell and Wright explained nimbers and gave a nim-addition table in "Wordnim and Grundyword".

In kayles, the values of chains of various lengths are as follows. Note that from length 71 onward, the values repeat in a cycle of 12; they are all shown in the bottom row of the table (see next page). It's clear that the first player (F) can always win: if the number of frags is even, F removes the middle two, otherwise F removes the middle one. The game then divides into two equal components. Now F uses a "copycat" strategy: whatever the second player (S) does in one component, F copies it in the other. This strategy can always be used when each chain length occurs an even number of times.

	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	1	4	3	2	1	4	2	6
12	4	1	2	7	1	4	3	2	1	4	6	7
24	4	1	2	8	5	4	7	2	1	8	6	7
36	4	1	2	3	1	4	7	2	1	8	2	7
48	4	1	2	8	1	4	7	2	1	4	2	7
60	4	1	2	8	1	4	7	2	1	8	6	7
72, 84...	4	1	2	8	1	4	7	2	1	8	2	7

A way to make the game more interesting is to start with two or more ana-gram-mar chains. Dudeney discusses the game starting from a row of 13 skittles of which the second is already down. Thus the chain lengths are 1 and 11. Suppose F and S play a corresponding position in word kayles, for example LONE ---- WORD BOOK SHOP MARK SHOT STAR RING LEAD BURN SIDE WAYS. The table above shows that the values of the chains are 1 and 6. F cannot convert the value 1 to 6, and so must split the long chain of value 6 into two chains whose nim-sum is 1. Their lengths must be 3 and 7. F can now take the lead by taking the LEAD. Now the chains are of length 1,7,3 and values 1,2,3.

Suppose that S now takes MARKSHOT. Now the chains are of length 1,3,2,3 and values 1,3,2,3, so their nim-sum is 3.

F cannot convert the value 3 to 0, so he converts the value 2 to 1 by taking STAR. The position now consists of two identical halves, so F now wins by the "copycat" strategy.

Word kayles can also be played in misere form, where the player who takes the last frag loses. If the game starts with one chain, S can force a win if its length is 1, 4, 9, 12 or 20, and F can force a win for all other chain-lengths up to 60.

If the rules of kayles are changed so as to require that the ball must knock down two skittles, we have a game which T.R. Dawson described in the Dec 1934 Fairy Chess Review. Elwyn Berlekamp, John Conway and Richard Guy in *Winning Ways* (Academic Press 1982), so I call the word version Dawsonword.

Now the stockpile of allowable words is just those made from two successive frags, without the single frags. The values are as follows. For chain lengths of 53 or more, the values repeat in a cycle of 34; they are as shown in the bottom two rows of the table.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	0	1	1	2	0	3	1	1	0	3	3	2	2	4	0	5
17	2	2	3	3	0	1	1	3	0	2	1	1	0	4	5	2	7
34	4	0	1	1	2	0	3	1	1	0	3	3	2	2	4	4	5
51	5	2	3	3	0	1	1	3	0	2	1	1	0	4	5	3	7
68,102...	4	8	1	1	2	0	3	1	1	0	3	3	2	2	4	4	5
85,119...	5	9	3	3	0	1	1	3	0	2	1	1	0	4	5	3	7

Dawsonword is interesting even when the game starts with a single chain of odd length. There are now chain lengths of value 0, which means that Dawsonword starting with a chain of such a length is a win for the second player.

For example, suppose F and S start with the chain of length 13 WORD GAME BALL WEED HOOK LIKE WISE LING BIRD LAND SHIP SIDE WAYS. The only good moves for F are to leave two chains of lengths 8 and 3, so F takes WEEDHOOK.

Suppose S takes WISELING. Now LIKE can never be taken, so in effect there are two chains of lengths 3 and 5. Their values are 1 and 0. Therefore F removes the short chain by taking WORDGAME, leaving a chain of length 5. Then, whatever S plays, F makes the last move and wins.

Dawsonword can also b played in misere form. As with the misere forms of other nim-like games, values of positions are not all nimbers. If the game starts with one chain, the chain-lengths which enable S to force a win are 2,3,7,8,12,16,17,21,22,26,30,31,35,36,40,45 and so on. If there is periodicity here, I haven't found it.

What if the players start with a chain of n frags, and each player must remove three successive frags? It turns out that they are playing a game very similar to Treblecross on a line of n-2 squares. Each frag, apart from the end ones, corresponds to a square. There is the small difference that putting a cross next to, or next but one to, an existing cross is illegal (rather than stupid). Such a move could also be described as putting a cross in, or next to, a square which is at, or next to, an existing cross. Putting a cross in a square corresponds to removing the frags corresponding to that square and the two squares next to it. An end square has only one neighbour, but putting a cross there must correspond to removing three frags in the frag game. So it is necessary to have an extra frag at each end.

APPENDIX

Players wanting long games will want long chains. An ana-gram-mar chain of 112 2-letter frags (Aug 1991 Word Ways, page 157) is fine for Dawsonword, where frags needn't be words. I have not found any ana-gram-mar chains in Word Ways longer than 22 frags with all frags different and with each frag being a word. Here are two, based mainly on solid words in Webster's Second with a few additions from Chambers. I do not pretend that they are anywhere near maximal, even with the constraint that all the frags be of the same length.

was her eon ism dom nei per ule tic ket one how kit ten son net cha kra sis kin cob web bed ull ing row let off ice cap tor rid den gue mal fed dan aid ant hem era ser dab ber gut tie pin ked ger min now hat red are ach ree sty lar don key way lay man nan dow set tee thy sen

dee pen sum pit chi nar ras cal low men tum mel lit hia tal war ran che lys sic sac rum pad nag ara lie nee bor rel ink pot leg end ear bob cat eye bar bel dam mar cor ona gra ben ami din gar rot tan tra gal ban tam ale san cho ana qua sky bal lam mas cot win das tur gor bet oil can non par sec ret ama mau met tar sal lee wan gan try pan tun dun lin aga mid rib bon sai the tch ast alk oxy gas bag ani cut out sin awa lim bat lan sat rap his ton sil van dal eth ane led (187 frags)

pina fore milk maid ling bird like wise weed hook heal able gate ward room mate lote bush wife hood mold made line ally late some what ever more fold boat bill fish yard land whin chat wood pile worm root fast hold back bite wing span king bolt rope walk mill race mule foot sore head rail road side hill sale work week long beak iron shot star lite rose drop seed case book rest rive rain wash outs hove ring dove tail pipe stem post cart boot lick spit fire plug tree nail shop folk free hand clap trap rock hair hoof mark down turn hall ower word play mare chal cone nose burn over past rami corn bell wind blow cock crow step sire ship load less ness (131 frags)